Roller Coaster (AP) Physics

Abridged Edition

An Abridged Educational Guide To Roller Coaster Design and Analysis

This resource booklet goes with an final AP physics project.

by Tony Wayne

INTRODUCTION

This booklet will discuss some of the principles involved in the design of a roller coaster. It is intended for the middle or high school teacher. Physics students may find the information helpful as well. Many of the concepts can be applied to topics other than roller coasters. Some sections will use the "Roller Coaster Simulator," RCS. (See page 78 for instructions on its construction.) The included activities are hands on cookbook type. Each section includes background topics that should have been taught previously.

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Calculation Algorithm to calculate a change in velocity associated with a change in height

Step 1 Identify two locations of interest. One with both a speed and a height and the other location with either speed or height.

Step 2 Write an equation setting the total energy at one location equal to the total energy at the other location.

Step 3 Solve for the unknown variable.

Example 1

What is the velocity at the bottom of the first hill?

Solution:

$ET_{(TOP)} = ET_{(BOTTOM)}$

KE + PE = KE +PE

$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$

$(1/2)v^2 + gh = (1/2)v^2 + gh$......... The masses cancel out because it is the same coaster at the top and bottom.

$(1/2)v^2 + gh = (1/2)v^2 + gh$............. Substitute the numbers at each location

$(1/2)(8.8)^2 + 9.8(95) = (1/2)v^2 + 9.8(0)$....... The height at the bottom is zero because it is the lowest point when comparing to the starting height.

$38.72 + 931 = (1/2)v^2$

$969.72 = (1/2)v^2$

$1939.44 = v^2$

$\therefore$ <u>v = 44.04</u> m/s .... at the bottom the the 1st hill.

Example 2

What is the velocity at the <u>top of the second hill</u>?

Solution:

$ET_{(TOP\ OF\ 1^{st}\ HILL)} = ET_{(TOP\ OF\ 2^{nd}\ HILL)}$

KE + PE = KE +PE

$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$

$(1/2)v^2 + gh = (1/2)v^2 + gh$................ The masses cancel out because it is the same Coaster at the top and bottom.

$(1/2)v^2 + gh = (1/2)v^2 + gh$................... Substitute the numbers at each location

$(1/2)(8.8)^2 + 9.8(95) = (1/2)v^2 + 9.8(65)$........... Notice all the numbers on the left side come from the top of the 1st hill while all the numbers on the right side come from the top of the 2nd hill.

$38.72 + 931 = (1/2)v^2 + 637$

$332.72 = (1/2)v^2$

$665.44 = v^2$

$\therefore$ <u>v = 25.80</u> m/s .... at the top the the 2nd hill.

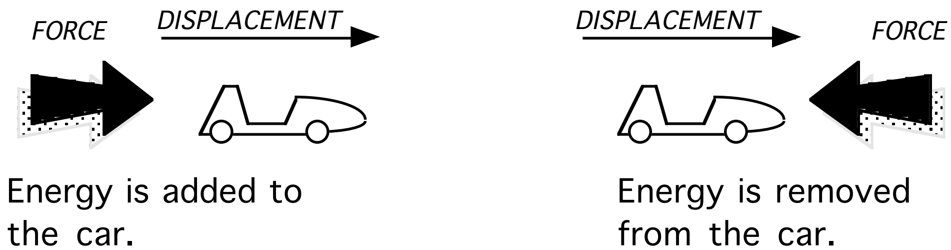
This technique can be used to calculate the velocity anywhere along the coaster.

Getting the Coaster Started

(Work, Kinetic Energy, Potential Energy , Kinematics and Power)

Getting The Coaster Started

Something has to be done to get the coaster started. In our previous example energy, power, has to added get the coaster up to 8.8 m/s. This is done by doing work on the coaster. A simplified definition of work would be force times displacement when the force and displacement go in the same direction. [This chapter will not go into all the details of calculating work.] Suffice it to say that when the force acting on the coaster and the displacement of the coaster are in the same direction, work adds energy to the coaster. When the force acting on the coaster and the displacement of the coaster are in opposite directions, work removes energy from the coaster.

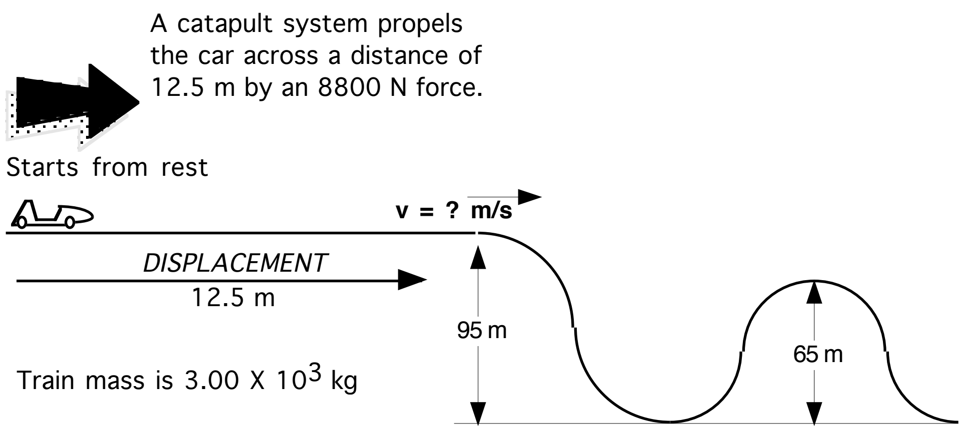


$$\text{Work} = (\text{Force})(\text{Displacement})$$
$$W = Fd$$

Where “work” is measured in joules, J. “Force” is measured in Newtons, N, and “displacement” is measured in meters, m.

Example 3

What is the velocity of the train after being catapulted into motion?



Solution:

$$ET_{(BEGINNING)} + \text{Work} = ET_{(TOP\ OF\ THE\ 1^{st}\ HILL)}$$
$$KE + PE + W = KE + PE$$
$$(1/2)mv^2 + mgh + Fd = (1/2)mv^2 + mgh$$
$$(1/2)3000(0)^2 + 3000(9.8)(0) + 8800(12.5) = (1/2)3000v^2 + 3000(9.8)(0)$$

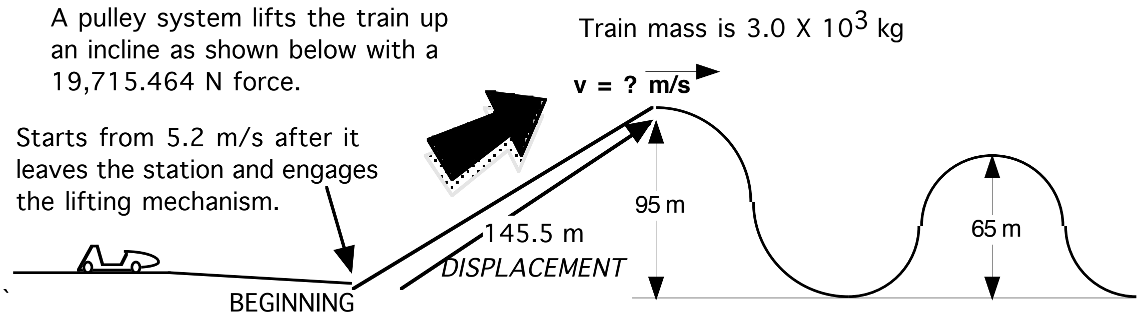
Substitute the numbers at each location

$$110{,}000 = (1/2)(3000)v^2$$
$$73.333 = v^2$$
$$v = 8.56\ m/s$$ ... at the end of the catapult.

As an aside you can calculate the acceleration of the rider from kinematics equations.
(For the curious the acceleration is 7.0 m/s^2)

Example 4

What is the velocity of the train after being hauled up a hill by chain driven motor system?

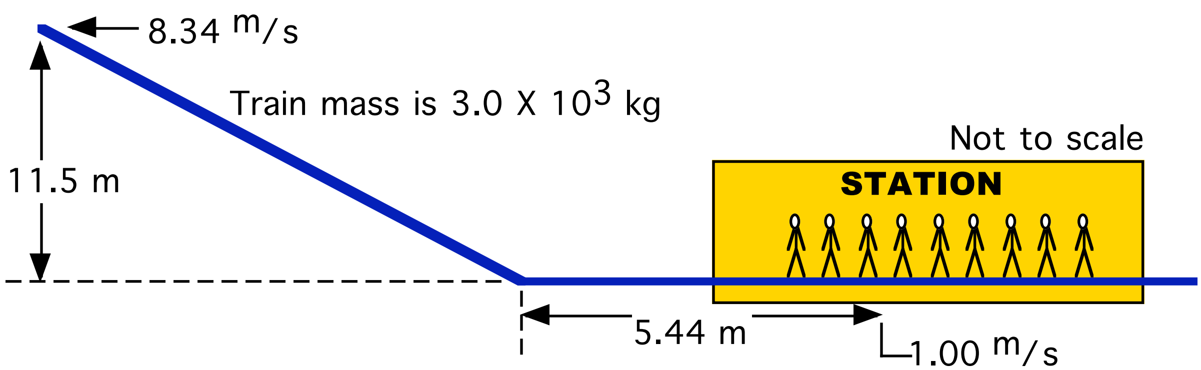


Solution:

$$ET_{(BEGINNING)} + Work = ET_{(TOP\ OF\ 1^{st}\ HILL)}$$
$$KE + PE + W = KE + PE$$
$$(1/2)mv^2 + mgh + Fd = (1/2)mv^2 + mgh$$
$$(1/2)3000(5.2)^2 + 3000(9.8)(0) + 19{,}715.464(145.5) = (1/2)3000v^2 + 3000(9.8)(95)$$
$$40560 + 2868600.012 = (1/2)(3000)v^2 + 2793000$$
$$116160.012 = (1/2)(3000)v^2$$
$$77.44 = v^2$$
$v = 8.8$ m/s ......at the top of the hill

Example 5

A roller coaster train of mass 3.0 X 10³ kg rolls over a 11.5 m high hill at 8.34 m/s before rolling down into the station. Once in the station, brakes are applied to the train to slow it down to 1.00 m/s in 5.44 m.



(a) What braking force slowed the train down?
(b) How much time did it take to slow the train down?
(c) What was the acceleration of the train in g's?

Solution:
(a)

$$ET_{(HILL)} = ET_{(@\ 1\ m/s)} + Work$$
$$KE + PE = KE + PE + W$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh + Fd$$
$$(1/2)3000(8.34)^2 + 3000(9.8)(11.5) = (1/2)3000(1)^2 + 3000(9.8)(0) + F(5.44)$$ Substitute the numbers

at each location

$$442433.400 = 1500 + 5.44F$$

F = 81053.9 N ...force to slow down the train

(b)

Calculate the velocity as the train enters the station. Use this velocity to calculate the time.

$ET_{(HILL)} = ET_{(@\ STATION\ ENTRANCE)}$ No work is done because no force other than gravity acts between the two locations

$$KE + PE = KE + PE$$

$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$

$$(1/2)3000(8.34)^2 + 3000(9.8)(11.5) = (1/2)3000v^2 + 3000(9.8)(0)$$

Substitute the numbers at each location

$$442433.400 = 1500v^2$$

$$v = 17.174$$

v = 17.2 m/s ...as the train enters the station.

The time is calculated from

$v_o = 17.174$ m/s

$v_f = 1.00$ m/s

$x = 5.44$ m

$t = ?$

$$\frac{x}{t} = \frac{v_o + v_f}{2}$$

$$\frac{5.4}{4} = \frac{17.174 + 1}{2}$$

t = 0.599 sec

This formula comes from combining two expressions for the average velocity. $\frac{x}{t} = v_{AVG}$ $\frac{v_o + v_f}{2} = v_{AVG}$

(c)

Calculate the acceleration in m/s^2. Then convert it into g's.

$F = ma$

$81053.934\ N = (3000\ kg)a$

$a = 37.018\ m/s^2$

$a = 37.018\ m/s^2 / 9.80\ m/s^2 = 3.78$ g's ... Yeow! That's a big jerk on the passengers into the restraining harness.

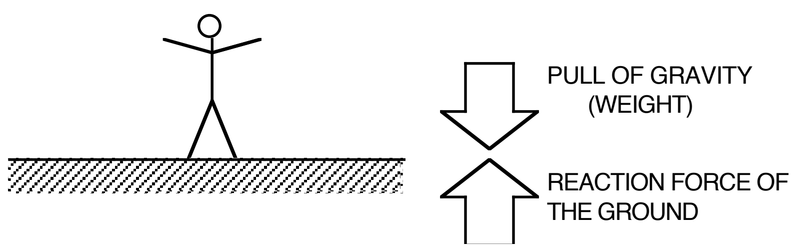
Weightlessness

WEIGHTLESSNESS

Weight is the pull of gravity. Typical weight units are pounds and newtons. (1 pound ≅ 4.45 newtons). On the moon, gravity pulls with 1/6 the force compared to the Earth. Therefore, a student on the moon weighs 1/6 of what she weighs on the Earth.
On the Earth, neglecting air resistance, all objects will speed up at a rate of 9.80 m/s every second they fall. That is a speed increase of about 22 mph for every second an object falls.

Time in the Air s	Velocity mph
0	0
1	22
2	44
3	66
4	88
5	110

There are two ways to experience weightlessness. (1) move far enough away from the planets and sun to where their pull is nearly zero. [Gravity acts over infinite distance. One can never completely escape it.] (2) Fall down at a rate equal to the pull of gravity. In other words, accelerate to the Earth speeding up 22 mph every second in the air. In order for a person to feel weight, a person must sense the reaction force of the ground pushing in the opposite direction of gravity.



In the absence of the reaction force a person will sink through the ground.
Many amusement park rides generate the weightless sensation by accelerating down at 22 mph every second.

g's

Neglecting air resistance, if a rock is dropped, it will accelerate down at 9.8 m/s^2. This means it will speed up by 9.8 m/s for every second it falls. If a rock you drop accelerates down at 9.8 m/s^2, scientists say the rock is in a "1 g" environment, [1 g = 9.8 m/s^2 = 22 mph/s].

Any time an object experiences the pull equal to the force of gravity, it is said to be in a "one g" environment. We live in a 1 g environment. If a rock whose weight on the Earth is 100 lbs was moved to a 2 g environment then it would weigh 200 lbs. In a 9 g environment it would weigh 900 lbs. In a "NEGATIVE 2 g" environment it would take 200 lbs to hold the rock down on the ground. In a "-5 g" environment it would take 500 lbs to hold the rock down to the ground. If the rock were put into a "zero g" environment then it would be weightless. However, no matter what happens to its weight the rock's mass would never change. Mass *measurement* is unaffected by the pull of gravity.

What does it feel like to walk in a 2 g environment? Have students find someone who's mass is about equal to theirs. Have them give piggyback rides. As they walk around this is what it feels like to be in a 2 g environment. Go outside on the soft ground and have the students step up on something. This is when they will really know what a 2 g environment feels like.

Often engineers will use g's as a "force factor" unit. The force factor gives a person a way of comparing what forces feel like.

All acceleration can be converted to g's by dividing the answer, in $^{m}/_{s^2}$, by 9.8 $^{m}/_{s^2}$.

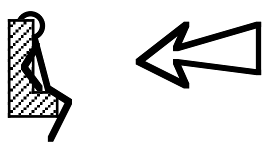
Example 6

A roller coaster is propelled horizontally by a collection of linear accelerator motors. The mass of the coaster train is 8152 kg. The train starts from rest and reaches a velocity of 26.1 $^{m}/_{s}$, 55 mph, in 3.00 seconds. The train experiences a constant acceleration. What is the coaster train's acceleration in g's?

Solution

$m = 8152$ kg
$v_o = 0$ (starts from rest)
$v_f = 26.1$ $^{m}/_{s}$
$t = 3.00$ s
$a = ?$

$v_f = v_o + at$
$26.1 = 0 + a(3.00)$
$a = 8.70$ m/s
in g's... 8.70 $^{m}/_{s^2}$ / 9.80 $^{m}/_{s^2}$ = <u>0.89 g's</u>
This means the rider is being pushed back into his seat by 89% of his weight.



The rider is pushed back into the seat by a force equal to 89 % of his weight.

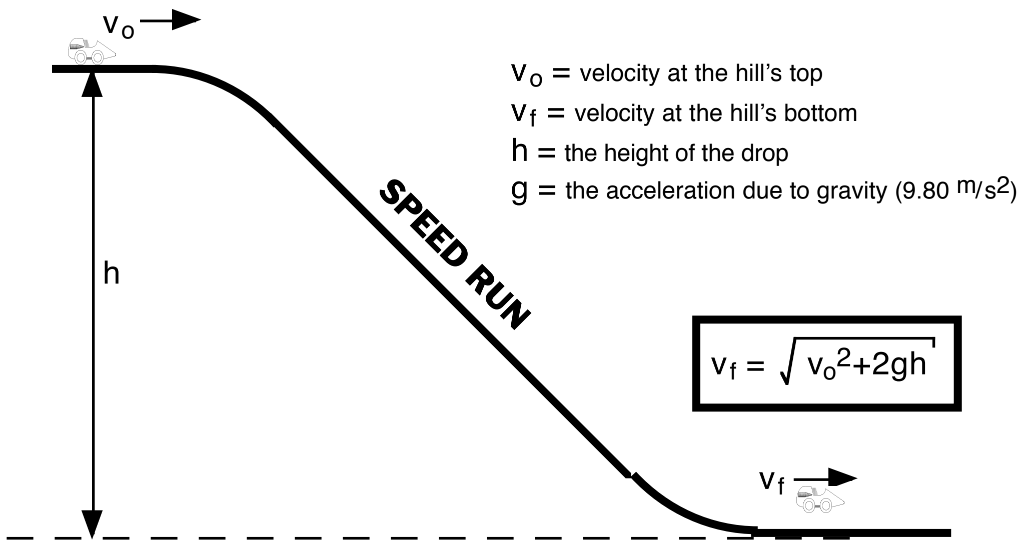
Hills and Dips

(Projectile Motion, Potential Energy and Kinetic Energy)

HILLS AND DIPS

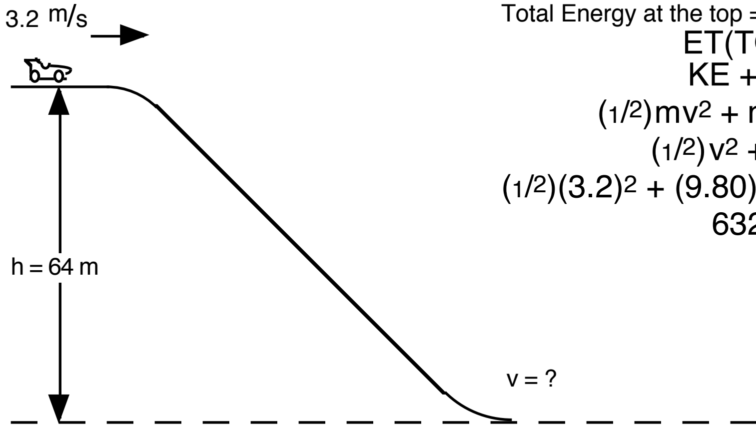
One of the most basic parts of a ride is going from the top of a hill to the bottom. There are two basics ways designs transport riders to the bottom of a hill. The first is called the “Speed Run.”

SPEED RUN DROPS



A speed run is designed to give the rider the feeling of accelerating faster and faster without the feeling of weightlessness. It simulates being in a powerful car with the accelerator held down to the floor. It is a straight piece of track that connects a high point to a low point.

The increase in velocity of the car comes from lost gravitational potential energy being converted into kinetic energy. Next to a horizontal straight piece, the speed run is the easiest piece of the track to design and analyze.



Total Energy at the top = Total Energy at the bottom

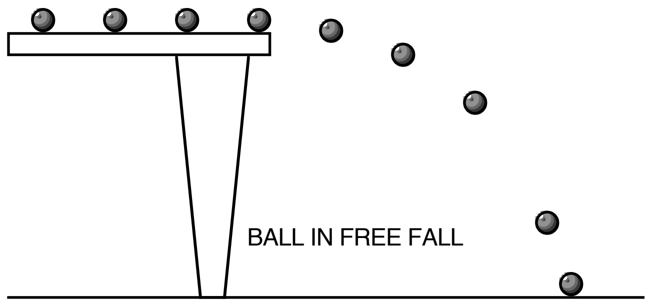
$$ET(TOP) = ET(BOTTOM)$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$$
$$(1/2)v^2 + gh = (1/2)v^2 + 0$$
$$(1/2)(3.2)^2 + (9.80)(64) = (1/2)v^2 + 0$$
$$632.32 = (1/2)v^2 + 0$$
$$v = 35.56$$
$$v = 36\ ^m/_s$$

When coasting up to a new height the calculations are the same as the example shown above. The shape of the hill does not matter. See the “Intro to Design” section, step 7, for an example of these up hill calculations.

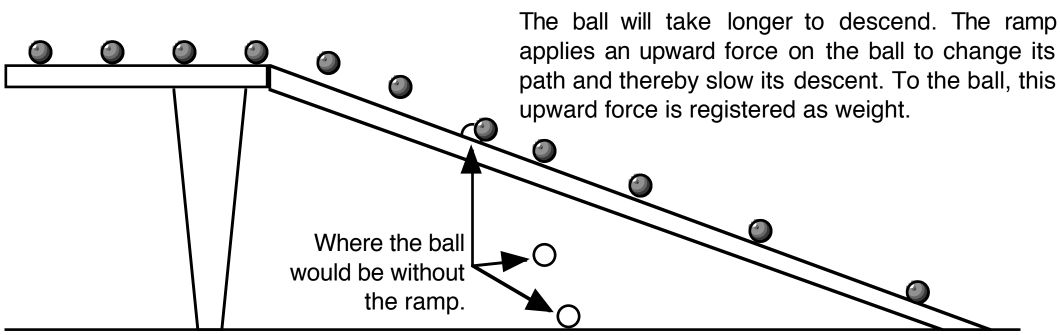
FREE FALL DROPS

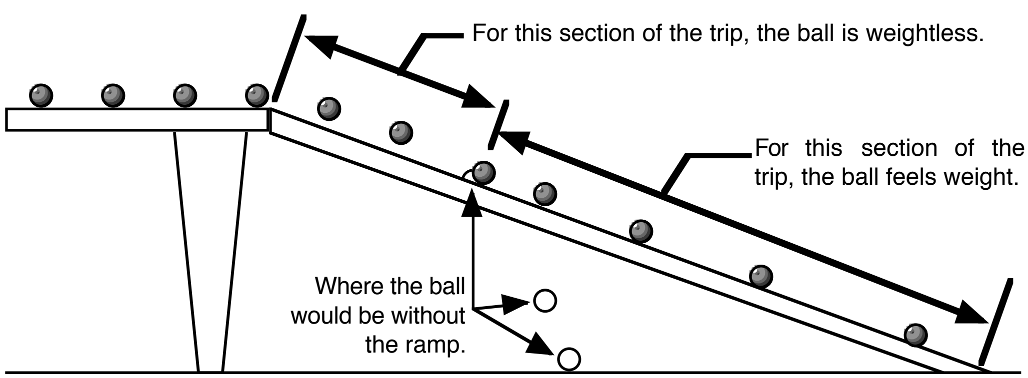
One of the biggest thrills on a roller coaster is the free fall as a rider travels over a hill. The easiest way to experience free fall is to hang from a tall height and drop to the ground. As a person falls he experiences weightlessness. As long as a person travels in the air like a projectile he will feel weightless.

Suppose a ball traveled off a table, horizontally, at 10 m/s. The ball's path would look like the path shown below.

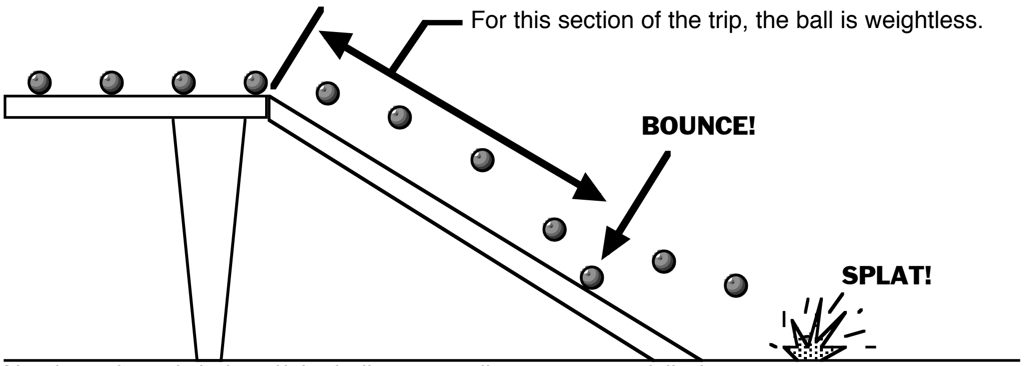


Now suppose the ball traveled off the table top on a shallow angled ramp. It would look like the one below.



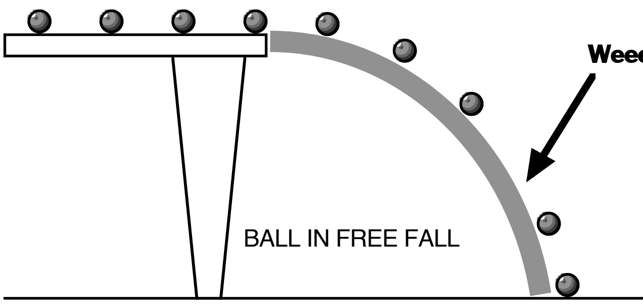


A straight, "speed run," drop does not match the fall of a rider over a hill.



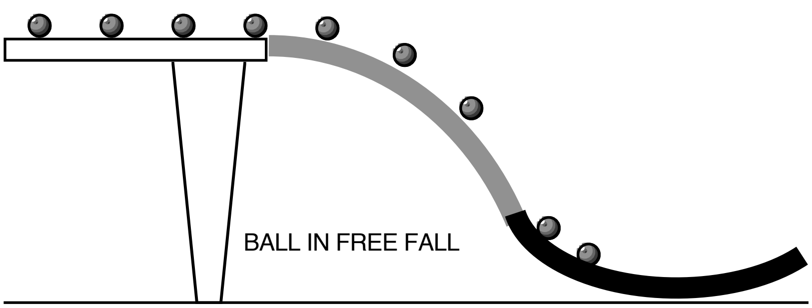
Not the safest of choices if the ball were a roller coaster car full of passengers.

To give riding more of a thrill, the designer needs to design the shape of the hill to match the falling ball.



The hill is the same shape as a projectile in free fall. The roller coaster barely makes contact with the track.

The only problem with curve above is the impact with the floor. To alleviate this problem another curve scoops the balls as they descend. This makes the ride smooth and survivable for the rider.



The speed at the bottom of a free fall drop is calculated the same way as the speed at the bottom of a speed run drop. The only difference is the shape of the hill from the top to the bottom.

PROJECTILE MOTION AND ROLLER COASTER HILLS

A free fall hill shape gives a rider a weightless sensation. To give this weightless sensation over a hill, the hill is designed to have the same shape as the path of a ball being thrown off the top of a hill. Shape is determined by how fast the roller coaster car travels over the hill. The faster the coaster travels over the hill the wider the hill must be. There are two ways to apply projectile motion concepts to design the hill's shape. The first way is to calculate the coaster's position as if it drove off a cliff.

The position equation is as follows.

$$h = \frac{gx^2}{2v^2}$$

h = the height from the top of the hill
x = is the distance away from the center of the hill
v = the velocity the roller coaster car travels over the top of the hill.
g = the acceleration due to gravity. 9.80 m/s² for answers in meters. 32.15 ft/s² for answers in feet.

This can be rewritten as

$$x = \sqrt{\frac{2hv^2}{g}}$$

EXAMPLE CALCULATIONS

Velocity (v) = 10 m/s
Acceleration (g) = 9.8 m/s^2

x in meters from hill's center	h in meters from the hill's top
0.00	0
4.52	1
6.39	2
7.82	3
9.04	4
10.10	5
11.07	6
11.95	7
12.78	8
13.55	9
14.29	10

Velocity (v) = 20 m/s
Acceleration (g) = 9.8 m/s^2

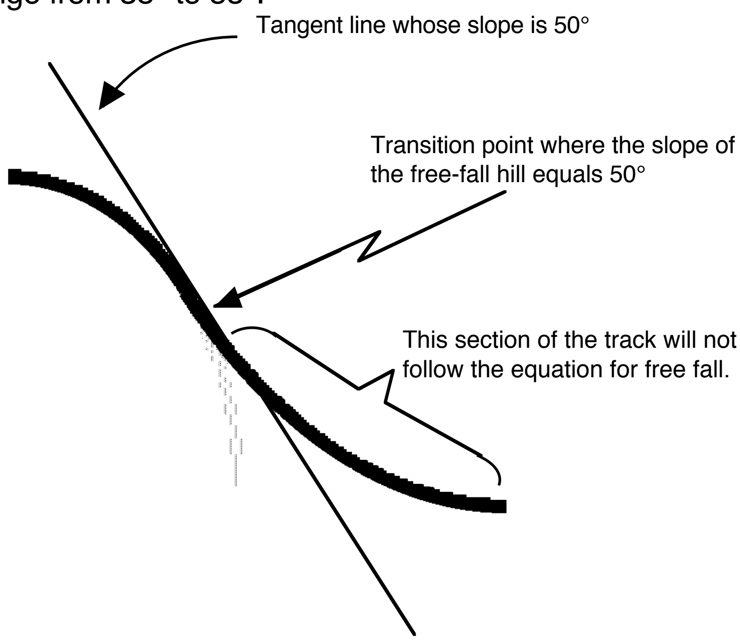
x in meters from hill's center	h in meters from the hill's top
0.00	0
9.04	1
12.78	2
15.65	3
18.07	4
20.20	5
22.13	6
23.90	7
25.56	8
27.11	9
28.57	10

These two tables use the second equation to calculate the position. For example, the calculation for line two in the v_o=10 m/s table looks like the equation below,

$$x = \sqrt{\frac{2\left(1m\right)\left(10\ \frac{m}{s}\right)}{9.80\ \frac{m}{s^2}}} = \boxed{4.52m}$$

Catching the Rider

There comes a certain point on the free-fall drop where the track needs to redirect the riders. Otherwise the riders will just plummet into the ground. This point is the transition point from free-fall to controlled acceleration. This point is also the maximum angle of a hill. This angle can be in virtually any range from 35° to 55°.



To calculate the angle of the hill use the same methods you would use to calculate the impact velocity of a projectile hitting the ground. In other words use the vetical and horizontal velocities to calculate the angle at that position.

Example:

Calculate the freefall's angle for a coaster then is traveling over the top at exactly 10 m/s, when the rider has dropped 4 meters from the crest of the hill.

Solution

To solve this the horizontal and vetical velocities at thei pointin space need to be calculated.

In the absence of any HORIZONTAL forces. the horizontal velocity will not change from the initial velocity. It is still 10 m/s.

The vertical velocity is found from kinematics.

$$\upsilon_{yo} = 0 \quad \text{...no initial vertical velocity}$$

$$\upsilon_y = o$$

$$h = 4m$$

$$a{=}g = 9.80\,\frac{m}{s^2}$$

$$\upsilon^2 = \upsilon_o^2 + 2gh$$

$$\upsilon_y = \sqrt{2gh}$$

$$\upsilon_y = \sqrt{2\left(9.80\,\frac{m}{s^2}\right)(4m)}$$

$$\boxed{\upsilon_y = 8.85\,\frac{m}{s}}$$

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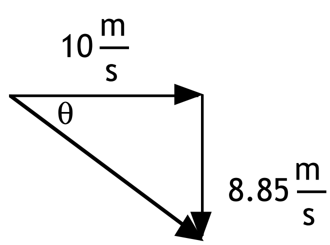


$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

$$\tan\theta = \frac{8.85\,\frac{m}{s}}{10\,\frac{m}{s}}$$

$$\boxed{\theta = 41.5°}$$

After dropping going over the top of the hill at 10 m/s nad dropping down 4 meters, the angle of the free fall shaped track is 41.5° beneath the horizontal. This kind of process lends itself well to a spreadsheet calculation.

For the bottom section of the track, the new equation has the desired outcome of changing the direction of the coaster from a downward motion to a purely horizontal motion. The track will need to apply a vertical component of velocity to reduce the coaster's vertical velocity to zero. The track will also need to increase the horizontal velocity of the coaster to the value determined from energy relationships. The velocity at the bottom of the hill is determined from

$$\underbrace{(\text{Kinetic Energy}) + (\text{Gravitational Potential Energy})}_{\text{Total mechanical energy at the top}} = \underbrace{(\text{Kinetic Energy})}_{\text{Total mechanical energy at the bottom}}$$

which is

$$(1/2)m(v_T)^2 + (mgh) = (1/2)m(v_B)^2$$

This simplifies to

$$v_B = \sqrt{(v_T)^2 + 2gh}$$

where v_B is the horizontal velocity at the bottom of the hill. The value for v_B will be used in later calculations.

Recall one of the original horizontal equations.

$$x = x_o + (v_{xo})t + (1/2)(a_x)t^2$$

substituting in our expression for "t" yields,

$$x = (v_{xo})\left(\frac{\sqrt{v_{yo}^2 + 2(a_y)y} - v_{yo}}{(a_y)}\right) + \frac{a_x}{2}\left(\frac{\sqrt{v_{yo}^2 + 2(a_y)y} - v_{yo}}{(a_y)}\right)^2$$

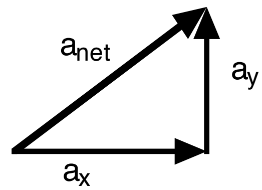
where v_{xo} is the horizontal velocity of the coaster at the transition angle and v_{yo} is the vertical component of the velocity at the transition angle. "a_y" is calculated from

$$v_y^2 = v_{yo}^2 + 2(a_y)y$$

and

$$(a_y) = \frac{v_y^2 - v_{yo}^2}{2y}$$

Where "v_y" is the final vertical velocity of zero, "v_{yo}" is the vertical component of the velocity at the transition point, and "y" is the distance left to fall from the transition point to the ground. The horizontal velocity is determined from a parameter decided upon by the engineer. The engineer will want to limit the g forces experienced by the rider. This value will be the net g's felt by the rider. These net g's are the net acceleration.



$$\therefore a_{net}^2 = a_x^2 + a_y^2$$

and

$$a_x = \sqrt{a_{net}^2 - a_y^2}$$

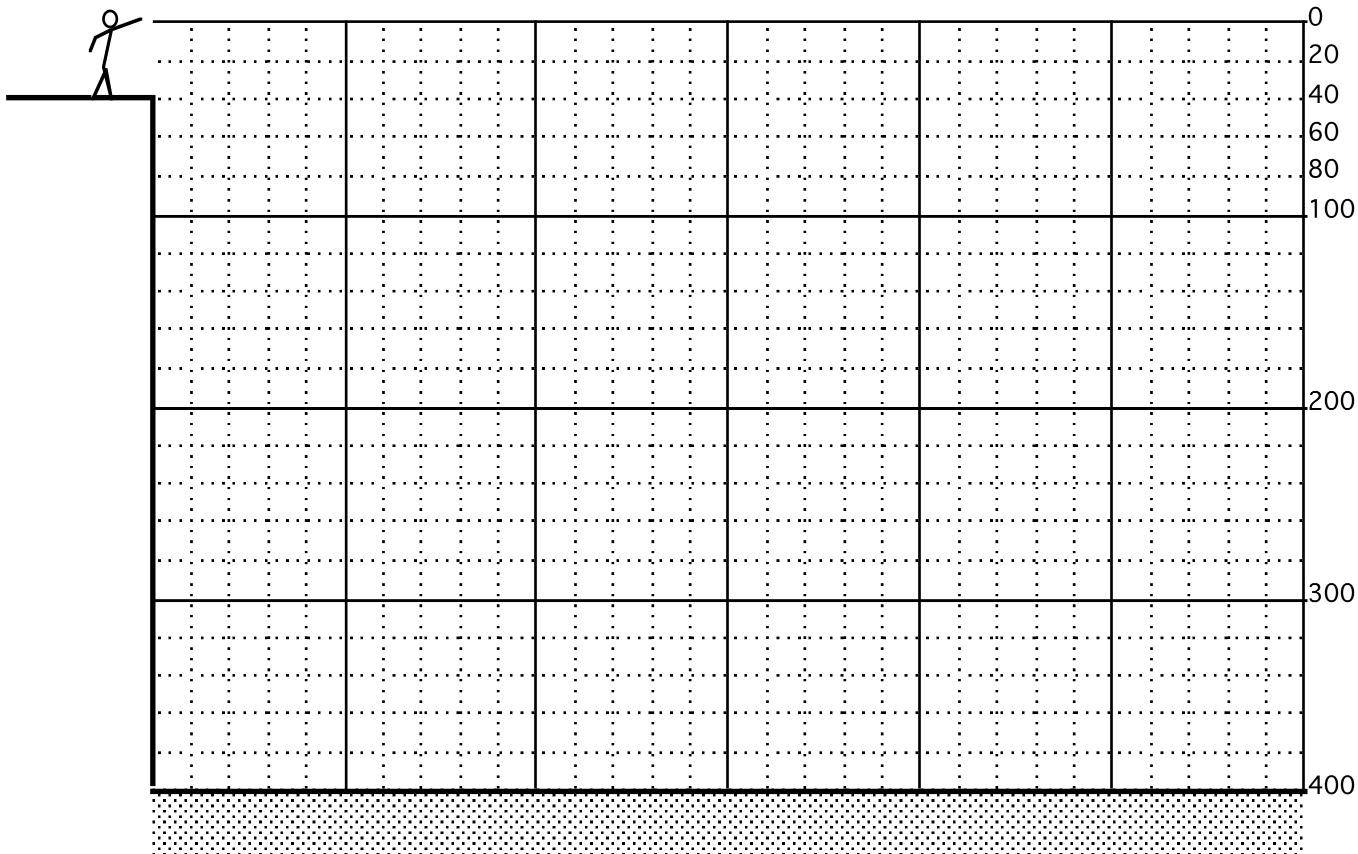
these values are plugged back into the original equation and x values are calculated as a function of y.

This is the beginning of the Hurler at Paramount's Kings Dominion in Doswell Virginia. Can you tell which hill is the free fall hill?

(It's the curved hill in front.)

Projectile Motion and Free Fall Hills Worksheet

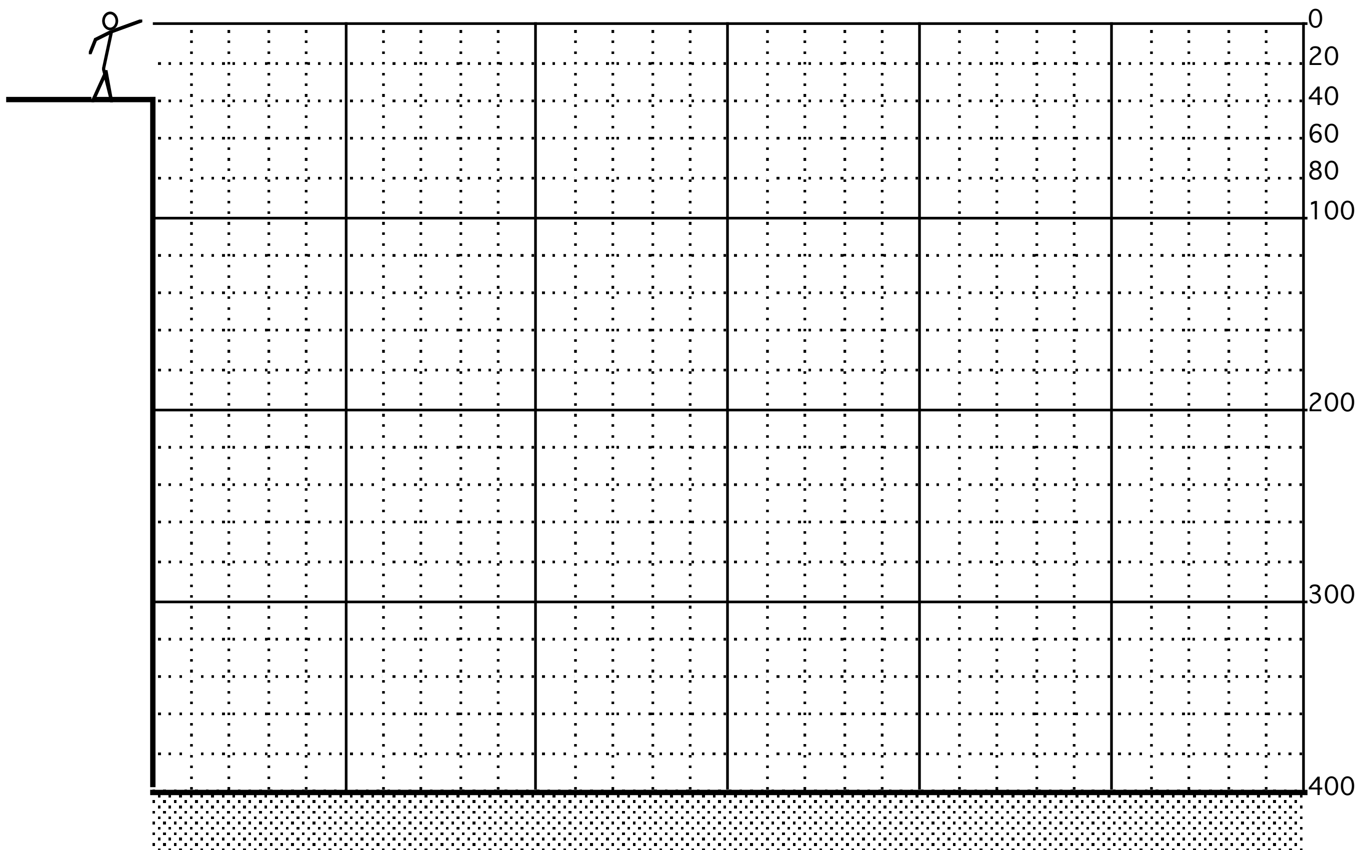
A person throws 2 balls. The first ball is thrown horizontally at 20 m/s. The second ball is thrown at 40 m/s. Draw as much of each path as possible. Draw the ball's position every 20 m of vertical flight. Draw a smooth line to show the curve's shape.



Vertical distance down (meters)	Horizontal position for the ball with an initial velocity of 20 m/s	Horizontal position for the ball with an initial velocity of 40 m/s
0	0	0
40		
80		
120		
160		
200		
240		
280		
320		
360		
400		

Projectile Motion and Hills Worksheet#2

A person throws 2 balls. The first ball is thrown horizontally at 10 m/s. The second ball is thrown at 30 m/s. Draw as much of each path as possible. Draw the ball's position every 1 second of the flight. Draw a smooth line to show the curve's shape.



Time (seconds)	Vertical position for the ball with an initial velocity of 10 m/s	Vertical position for the ball with an initial velocity of 30 m/s
0	0	0
1		
2		
3		
4		
5		
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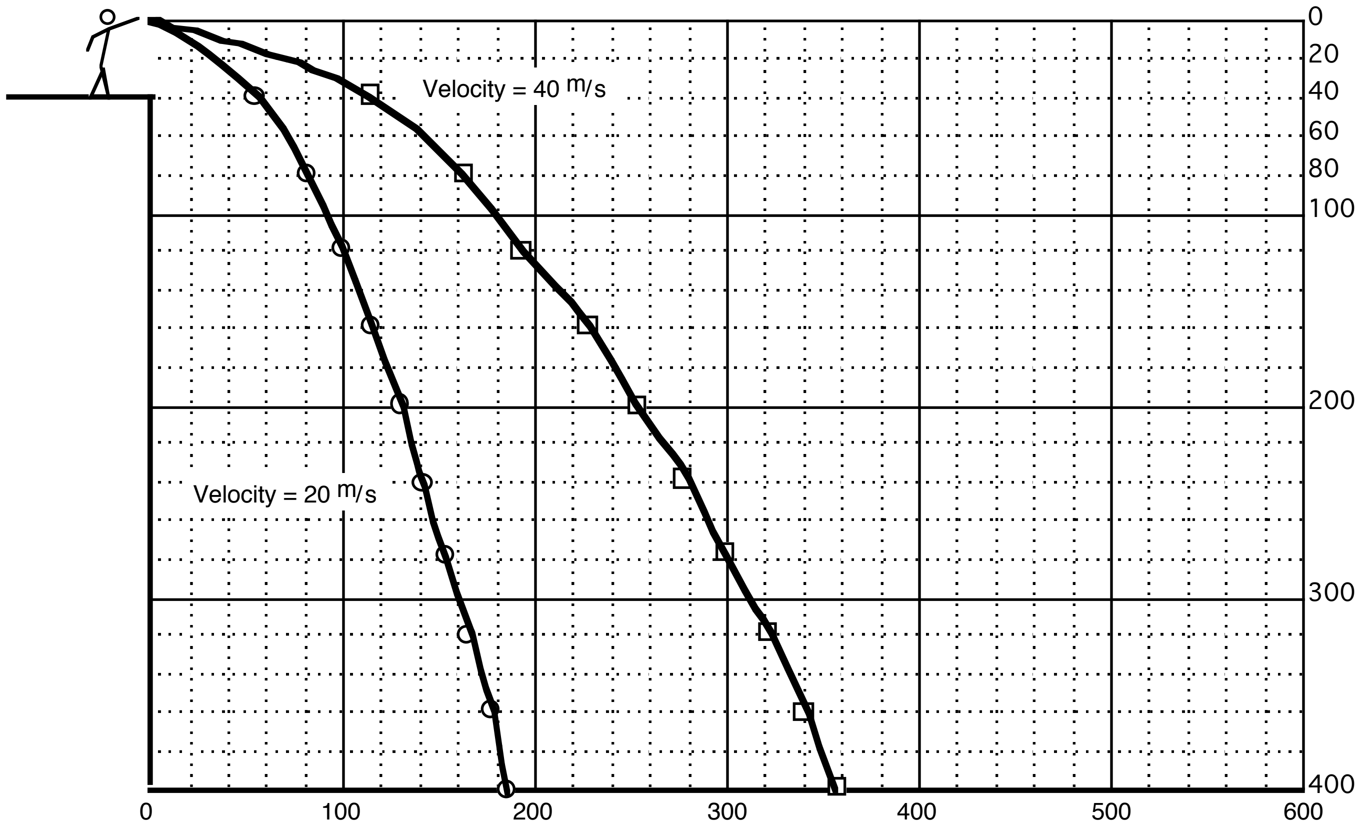
Projectile Motion and Free Fall Hills Worksheet
ANSWERS

A person throws 2 balls. The first ball is thrown horizontally at 20 m/s. The second ball is thrown at 40 m/s. Draw as much of each path as possible. Draw the ball's position every 20 m of vertical flight. Draw a smooth line to show the curve's shape.



Below is a set of answers for 4 velocities.

Vertical distance down (meters)	Horizontal position for the ball with an initial velocity of 10 m/s	Horizontal position for the ball with an initial velocity of 20 m/s	Horizontal position for the ball with an initial velocity of 30 m/s	Horizontal position for the ball with an initial velocity of 40 m/s
0	0	0	0	0
40	28.6	57.1	102.4	136.6
80	40.4	80.8	121.8	162.4
120	49.5	99.0	134.8	179.8
160	57.1	114.3	144.9	193.2
200	63.9	127.8	153.2	204.3
240	70.0	140.0	160.3	213.8
280	75.6	151.2	166.6	222.2
320	80.8	161.6	172.3	229.7
360	85.7	171.4	177.4	236.6
400	90.4	180.7	182.2	242.9

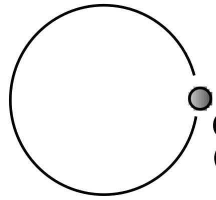
Loops

(Circular Motion, Potential Energy and Kinetic Energy)

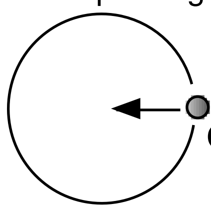
LOOPS

CENTRIPETAL FORCE

The average person on the street has heard of centrifugal force. When asked, they would describe this force as the one pushing an object to the outside of a circle. There is only one problem with this description. There is NO FORCE pushing an object to the outside.



For a person riding in a car while traveling in a circle, he perceives a force pushing him to the outside of the circle. But what force is physically pushing him? It can not be friction. Frictional forces oppose the direction of motion. It can't be a "normal force[1]." There is not a surface pushing the rider to the outside. To travel in a circle, a force pointing to the inside of the circle, or curve, is needed. The force pointing to the inside is called the *centripetal force*.

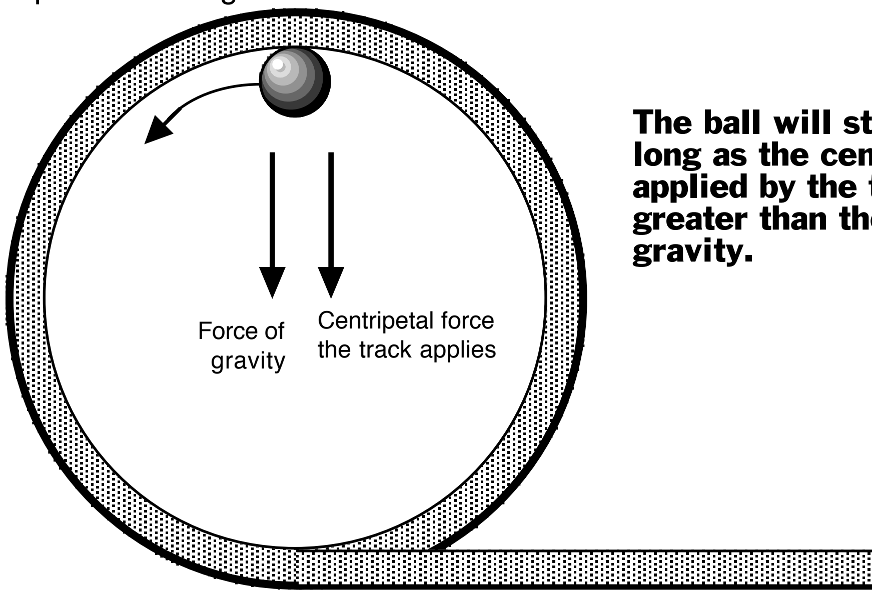


To understand a source for the misconception of the direction of this force, consider what it feels like when traveling around a corner in the back seat of a car. Everyone who has been in this situation knows that the passenger will slide to the outside of the curve. To understand that there is no force pushing the passenger to the outside, a change of reference frame is needed. Move the point of view from inside the car to a location outside, above, the car.

[1] The normal force is the force perpendicular to a surface. The floor is exerting a normal force straight up equal to your weight right now.

A COASTER'S LOOP

On a well designed roller coaster loop, the riders will not be able to sense when they are traveling upside down. This is done by making sure the force that is exerted on the rider is at least equal to the weight of the rider.



The ball will stay on the track as long as the centripetal acceleration applied by the track is equal to or greater than the acceleration of gravity.

Centripetal force applied to the track depends on the velocity of the car and inversely to the radius. The formula is:

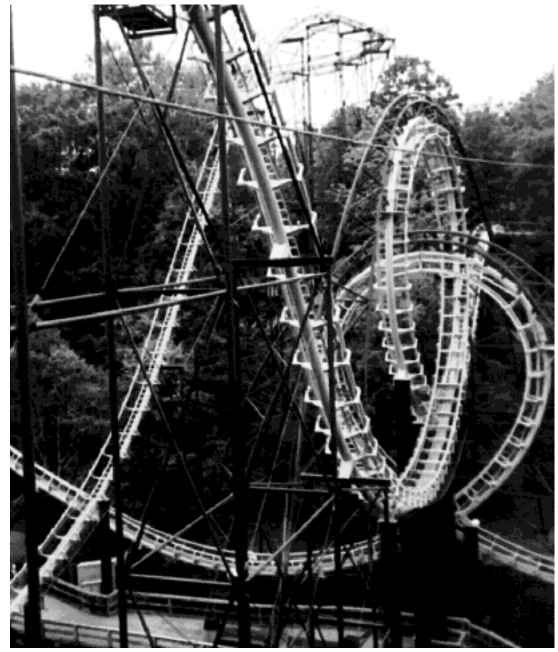
$$F = m\frac{V^2}{R}$$

F = Centripetal force
m = mass of the object going in a circle
v = Object's velocity
R = Radius of circle of curve
a_c = centripetal acceleration

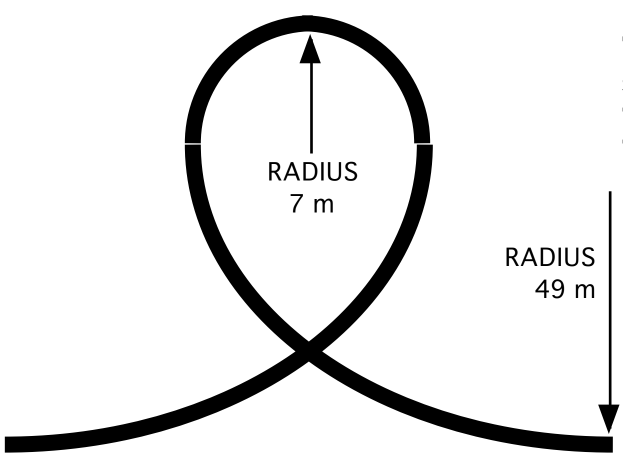
$$a_c = \frac{V^2}{R}$$

In order to apply enough centripetal acceleration the roller coaster car has to either be traveling very fast or the radius of the loop has to be made small. Most rides have a tall loop. A tall loop means a big radius. The problem is, as a car goes up, it slows down. The higher it goes, the slower it will be traveling over the top. In order to apply a centripetal force equal to gravity, 1 g, at the top, the car must be traveling extremely fast as the rider enters the loop. On some of the early round loops, the riders actually had their necks broken as a combination of the sudden rise in the loop as they entered at an extremely high rate of speed. As a compromise, the loops today are designed around an irregular shape called a klothoid or spiral of Archimedes. These irregular loops allow a circular figure whose radius changes.

"Klothoid" shaped loop from the Shock Wave at Paramount's Kings Dominion in Doswell, Virginia.



This is the Loch Ness Monster at Busch Gardens in Williamsburg, Virginia. It has two loops that are designed from the spiral of Archimedes. One loop is easy to identify in the picture. Can you spot the second loop?



Klothoid or Spiral of Archimedes Loop

The loop is tall and the radius is small. The smaller radius means the coaster car can travel slower and still apply enough force so the riders do not fall out.

For the advanced reader, the formula for the klothoid shape is:

$$x = \pm A \int_0^t \frac{\sin(t)\,dt}{\sqrt{t}} \qquad y = \pm A \int_0^t \frac{\cos(t)\,dt}{\sqrt{t}}$$

Asymptotic points: $(\pm A/2, \pm A/2)$

The formula for the "Spiral of Archimedes" in polar form is

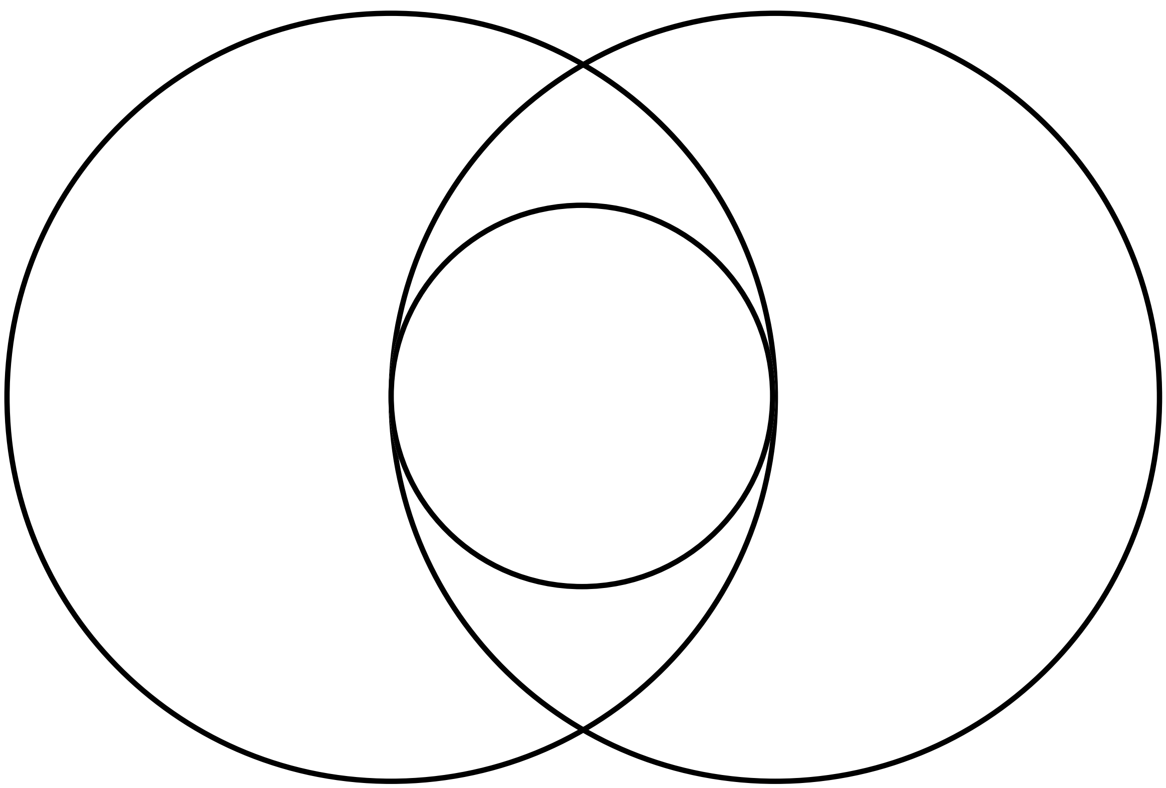
$$\mathbf{r = a\theta}$$

where "a" describes the magnitude of the spiral and "θ" is the angle through which the spiral is formed. To make a loop, the spiral will have to be mirrored horizontally.

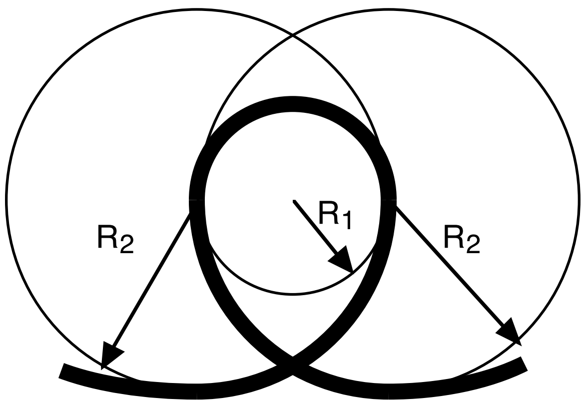
Nothing is perfect in engineering. These designs operate under ideal circumstances. In real life, the curves need to be tweaked into the right shape.

The Simple Irregular Loop

Sometimes it is not necessary to go into all the math to have a little fun with the irregular loop. These loops can be simulated using the combination of semi-circles of different radii.



Can you see the irregular loop in these regular circles?



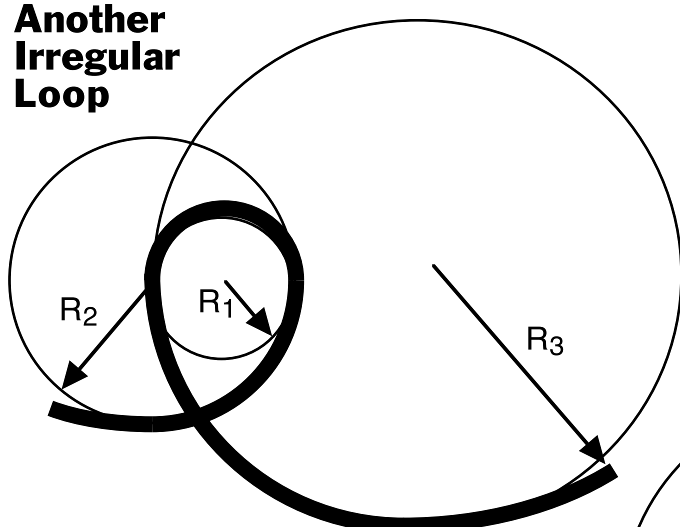
The radii can be anything as long as the car will make it around. In this particular drawing the height at the top of the loop from the very bottom is $(1/2)R_2 + R_1$.

If the engineer so chose, she could make the radius at the bottom on the way in one value and the bottom radius on the way out a different value.

Do not design a real roller coaster with this method. The transition from different radii would be uncomfortable for the rider and not possible for the roller coaster train.

Other loop possibilities

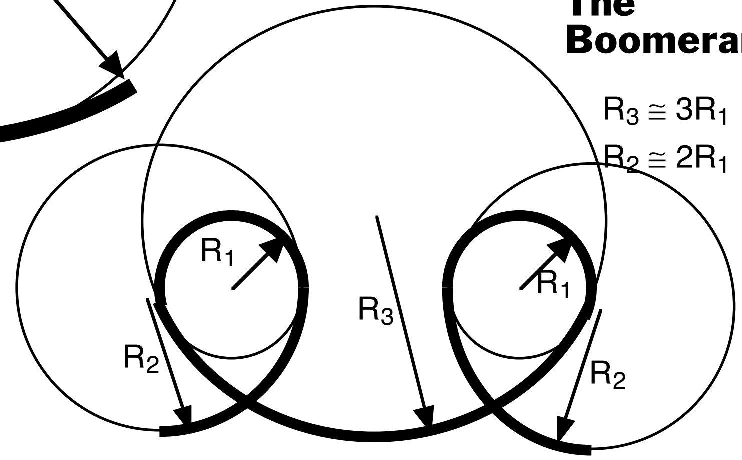
Another Irregular Loop



The Boomerang

$R_3 \cong 3R_1$

$R_2 \cong 2R_1$

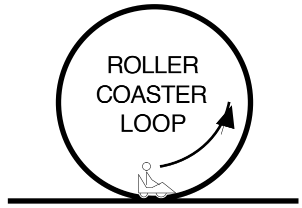


Physiological Effects of Acceleration

(Acceleration, Circular Motion, Kinetic Energy, and Potential Energy)

BACKGROUND

Imagine a passenger riding through a loop on a roller coaster. The passenger's head is towards the inside of the circle.



Her feet are to the outside of the circle. In order to keep blood in the passenger's head, a centripetal force needs to be applied to the blood to push it upwards toward the head and the center of the circle. The heart applies the centripetal force on the blood. A passenger can experience many g's in a loop. Recall that a g is the number of times heavier an object becomes. A 7 g experience means that the passenger feels 7 times heavier. Everything about the passenger becomes 7 times heavier. Her 3 pound brain now weighs 21 pounds. Every ounce of blood becomes 7 times heavier. If the blood feels too heavy the heart cannot apply enough force to push it towards the head. If the brain does not get any blood it will not get the oxygen the blood carries. The passenger will pass out within a second.

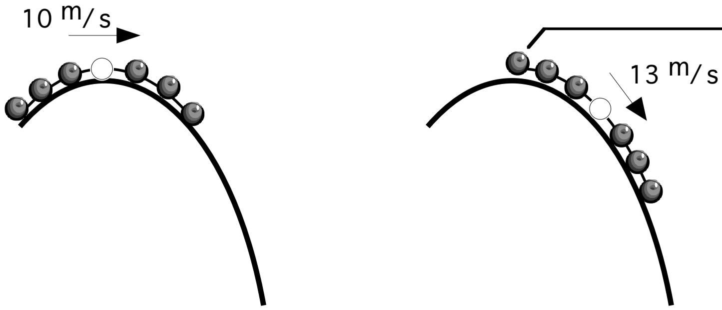
THE EXTREME EXPERIENCE

You are riding a new untested roller coaster when something goes wrong. As you enter the first big loop, a great pressure pushes you down. You slouch down in the seat from the extra weight. Over the top of the loop the roller coaster car slows down. The extra weight on your legs, lap, and shoulder make it impossible to sense that you are upside down. Out of the loop, over a hill and into another loop. This loop has a smaller radius. The car is traveling much faster now. As the g forces climb up toward 7 g's, you sink further still in the seat. You can no longer see color. Everything appears in black and white. An instant later, the passenger next to you disappears from view. Your field of vision is shrinking. It now looks like you are seeing things through a pipe. The front corner of the car disappears from view as your peripheral vision disappears. The visual pipe's diameter is getting smaller and smaller. You sink into the seat further still as the number of g's climb further. In a flash you see black. You have just "blacked out." You are unconscious until the number of g's are reduced and the blood returns to your brain.

Amusement park owners and insurance companies don't want the previously described situation to occur. It would limit repeat riders and the number of potential consumers who can safely ride the coaster. Most roller coasters keep the g's felt under 5 g's on an inside loop or the bottom of a dip after a hill. When a rider travels over a hill at a high rate of speed, he experiences negative g's. A negative g is the multiple of a person's weight that is needed to keep a rider in his seat. Negative g's also force the coaster car to try to come up off the track. Negative g's are a rider's heaven and a designer's nightmare. Negative g's are avoided as much as possible.

A negative g has a different effect on a rider than a positive g. Both negative and positive g's can cause a rider to pass out. But negative g's cause a rider to "red out." A red out condition occurs when there is too much pressure on the brain caused by too much blood in the head. The extra pressure can cause blood vessels to burst and kill the rider. This is a sure way to limit the number of repeat riders.

There is another way for a rider to experience negative g's. It is related to the length of the train. The roller coaster track is designed for the dynamics at the center of mass of the coaster train. Negative g's are experienced by the rider at the back of the train as he travels over a hill. For an empty train, the center of mass is in the middle of the train.Whatever speed is acquired by the center of the train is the speed for the entire train. After the center of a train passes over a hill it begins to gain velocity. As the center speeds up so does the back of the train. This means that the rear of the train will travel over the hill faster than the middle of the train. If the rider travels over the hill faster than the designed velocity of the hill the rear car will be whipped over the hill.

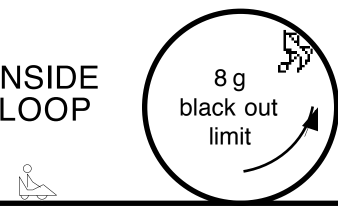


The rear car is traveling over the hill at 3 m/s faster than the center was. If the hill is designed for 10 m/s then the rear train car will have a tendency to leave the tracks. Under-carriage wheels will hold it to the tracks.

SOME "g" DETERMINATORS:

EFFECTS FOR THE AVERAGE HEALTHY PERSON

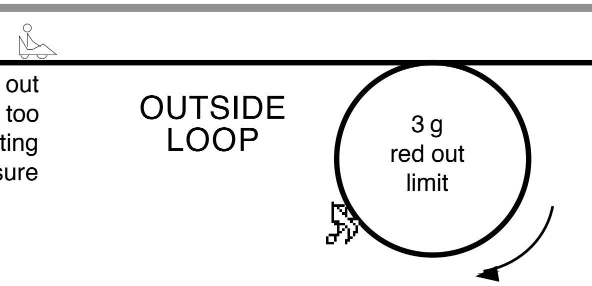
INSIDE LOOP



A person passes out because of the lack of oxygen in the brain.

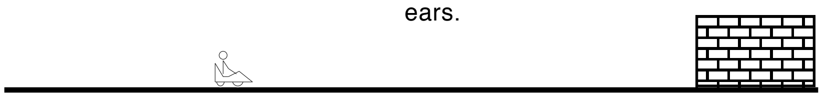
A person passes out because of too much blood creating too much pressure on the brain.

OUTSIDE LOOP



LINEAR ACCELERATION

20 g stress during a quick acceleration in the direction of motion. Bleeding would occur from ears.



LINEAR ACCELERATION

40 g stress during a quick acceleration in the direction of motion. Death at 40 g's.

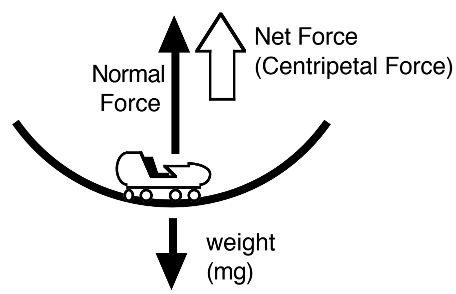
CALCULATION OF THE G'S FELT

To calculate the g's felt, a formula from circular motion will be utilized. Since energy relationships do not utilize time, the circular motion formula used will also not utilize time.

$$a_{CENTRIPETAL} = \frac{v^2}{R}$$

$$g\text{'s} = \frac{a_{CENTRIPETAL}}{9.8\ \frac{m}{s^2}}$$

Where "**v**" is the velocity of the body and "**R**" is the radius of the circle traveled. To calculate the velocity a body is traveling, use energy relationships to solve for the kinetic energy and the associated velocity. One more thing. To calculate the g's felt remember that the g's felt by the rider is the normal force on the seat of the rider divided by the mass then converted into g's. As a rider enters a loop he will feel 2 forces.



The real number of interest is the number if g's felt by the passenger traveling in the vertical circle. The g's felt are calculated below.

$\Sigma F_y = m(a_c) =$ (Normal Force) - Weight

$\Sigma F_y = mv^2/R =$ (Normal Force) - mg

$\therefore$ (Normal Force) $= mv^2/R + mg$

recall that... (Normal Force)/mg = g's felt by the rider

thus... (Normal Force)/g $= mv^2/R/mg + mg/mg$

$\therefore$ **g's felt by the rider = (centripetal acceleration in g's) + 1** at the bottom

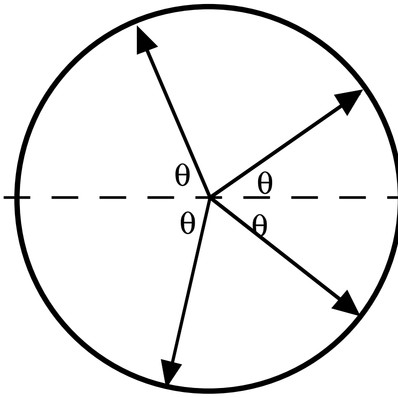
These results can be summarized as follows...

ABOVE the horizontal

$$\text{g's FELT} = \text{g's} - \sin(\theta)$$

$$\text{g's FELT} = \text{g's} + \sin(\theta)$$

BELOW the horizontal



ZERO degrees (Horizontal)

All angles are measured to the horizontal axis

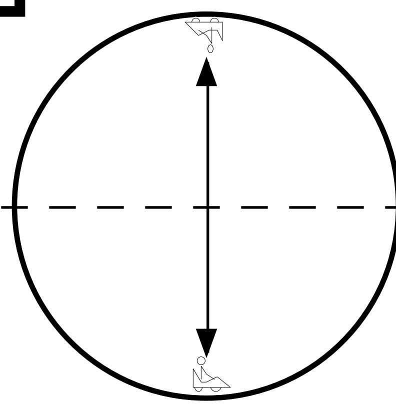
SPECIAL CASE (SHORT CUT)

ABOVE the horizontal

$$\text{g's FELT} = \text{g's} - 1$$

$$\text{g's FELT} = \text{g's} + 1$$

BELOW the horizontal



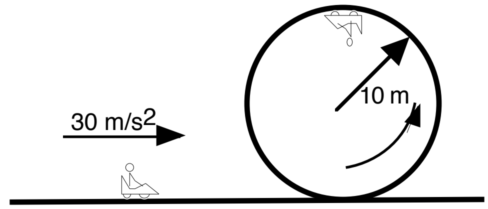
ZERO degrees (Horizontal)

All angles are measured to the horizontal axis

These results can be interpreted easily. As a rider enters the loop, the track has to exert a normal force upwards to supply the necessary centripetal force and acceleration to make the rider travel in a circle. But because the loop is vertical and the rider is at the bottom the normal force not only has to supply the centripetal force but must *also* overcome the pull of gravity. That's why 1 g is added in the equation. At the top of the loop, 1 g is subtracted from what is felt because the pull of gravity is helping the normal force exerted by the track instead of needing to be overcome.

EXAMPLE 1

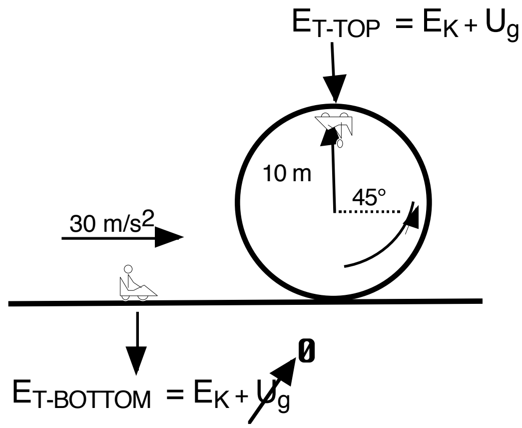
In a roller coaster ride a rider travels as shown to the right. How many g's will the rider feel at the top of the loop?



SOLUTION

To calculate the g's at the top of the loop, you will need to know the velocity of the rider there. To find velocity, use kinetic energy.

$$E_{T\text{-}TOP} = E_{T\text{-}BOTTOM}$$
$$E_K + U_g = E_K + 0$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$$
$$(1/2)v^2 + gh = (1/2)v^2$$
$$(1/2)v^2 + (9.8)17.07106 = (1/2)(30)^2$$
$$(1/2)v^2 + 167.2964 = (1/2)30^2$$
$$282.7036 = (1/2)v^2$$
$$565.4072 = v^2$$
$$\mathbf{v = 23.7783\ m/s}$$



$$a_{CENTRIPETAL} = \frac{(23.7783\ m/s)^2}{10} = \frac{565.4076}{10} = 56.5408\ m/s^2$$

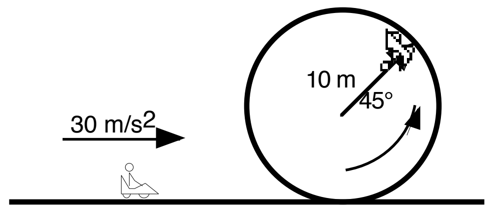
$$g\text{'s} = \frac{56.5408\ m/s^2}{9.8\ m/s^2} = 5.77\ g\text{'s}$$

g's FELT = 5.77-1 = **4.77 g's felt**

The rider will not pass out because 4.77 is less than 8 g's.

EXAMPLE 2

In a roller coaster ride a rider travels as shown to the right. How many g's will the rider feel at this location of the loop?



SOLUTION

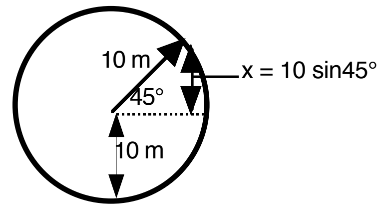
height of the roller coaster car;

$h = \text{radius} + x$

$h = 10 + 10\sin 45°$

$h = 10 + 7.07107$

$h = 17.07107$ m



$E_{T\text{-}TOP} = E_{T\text{-}BOTTOM}$

$E_K + U_g = E_K + 0$

$(1/2)mv^2 + mgh = (1/2)mv^2 + 0$

$(1/2)v^2 + gh = (1/2)v^2$

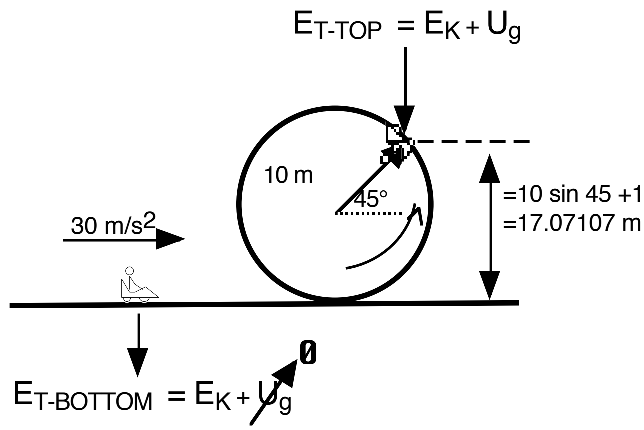
$(1/2)v^2 + (9.8)17.0711 = (1/2)(30)^2$

$(1/2)v^2 + 167.30 = (1/2)30^2$

$282.703 = (1/2)v^2$

$565.406 = v^2$

$\mathbf{v = 23.778\ m/s}$



$$a_{CENTRIPETAL} = \frac{(23.778\ m/s)^2}{10} = \frac{565.406}{10} = 56.541\ m/s^2$$

$$g\text{'s} = \frac{56.541\ m/s^2}{9.8\ m/s^2} = 5.77\ g\text{'s}$$

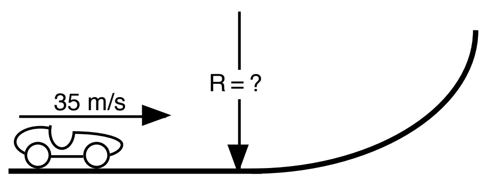
$$g\text{'s FELT} = 5.77 - \sin 45° = \mathbf{5.01\ g\text{'s felt}}$$

The rider will not pass out because 5.01 is less than 8 g's.

THE IRREGULAR LOOP EXAMPLE

The simple loop is easy enough to calculate. The irregular shaped loop needs a little more work. The velocity as the car enters the loop should be known. First establish the g's felt at the bottom. Subtract one g to know what the track exerts. Then convert these g's to m/s^2. Now solve for the radius.

EXAMPLE



STEP 1 (I'm randomly choosing 6 g's as the limit for the rider)
Therefore the centripetal acceleration of the track is 6g - 1g = 5g's.

STEP 2 (convert these g's to m/s^2)

$$(5g)\left(\frac{g}{9.80\ m/s^2}\right) = 49\ m/s^2$$

STEP 3

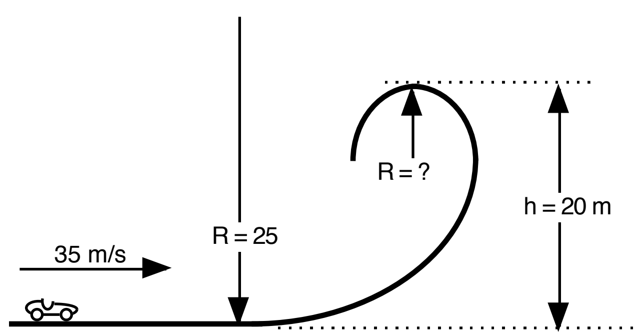
$$a = \frac{v^2}{R}$$

$$49\ m/s^2 = \frac{35^2}{R}$$

$$R = 25\ m$$

Now to calculate what the rider feels at the top of the loop.

Decide on the height of the loop. Then decide how many g's the rider will experience. Use the loop formulae with centripetal acceleration to calculate the radius.



STEP 4

The top of the loop will be at 25 m. (Chosen pretty much at random.)

STEP 5

I'm randomly choosing 6 g's again as the limit for the rider. It could be almost any number. At the top of the loop add 1 g for the centripetal force. ("Add" because the rider is upside down.)

6g's + 1 g = 7g

STEP 6 convert g' to m/s^2

$$(7g)\left(\frac{g}{9.80\ m/s^2}\right) = 68.6\ m/s^2$$

STEP 7

$(1/2)(m)(35)^2 = (1/2)(m)v_o^2 + (m)(9.80\ m/s^2)(20\ m)$

The m's divide out.

$(1/2)(35)^2 = (1/2)v_o^2 + (9.80\ m/s^2)(20\ m)$

$v_o = 28.86\ m/s$

STEP 8

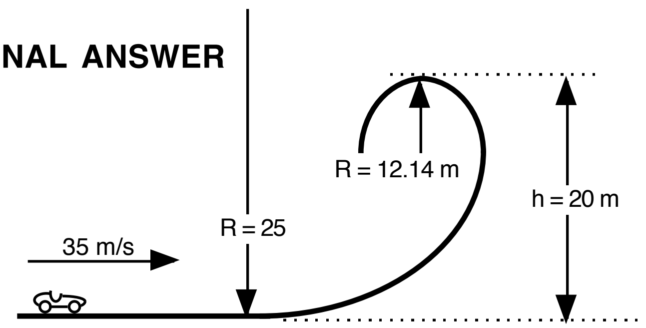
Calculate the radius at the top

$$a = \frac{v^2}{R}$$

$$68.6\ m/s^2 = \frac{28.86^2}{R}$$

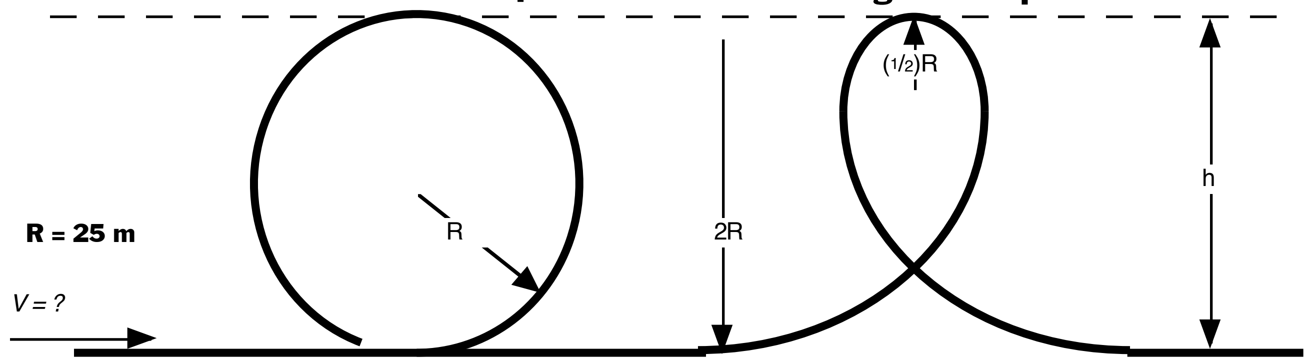
$R = 12.14$ m

FINAL ANSWER



REALITY CHECK

In reality, a person will not pass out the instant he/she reaches 8 g's. It will take a few seconds of being at 8 g's for the person to pass out. But for the sake of easy calculations we will assume that the instant 8 g's is reached a person will pass out. 8 g's is an average. peaple generally pasout between 6 to 10 g's. (FYI: The 40 g mark mentioned earlier is instantaneous for <u>death</u>.)



The height of both loops is the same.

1 What must the velocity of the car be at the top of the circular loop such that the rider FEELS weightless at the top of the first loop?

2 What must the velocity of the car be at the bottom of the circular loop such that the rider FEELS weightless at the top of the first loop?

3 How many g's does the rider feel as he enters the circular loop, at the bottom?

4 How fast is the rider traveling when he enters the irregular loop?

5 How many g's does the rider feel as he enters the irregular loop?

6 How fast is the rider traveling at the top of the irregular loop?

7 How many g's does the rider feel at the top of the irregular loop?

NEW SET OF PROBLEMS

8 How fast must the car be traveling at the top of the klothoid loop if the rider is to experience 2.00 g's?

9 How fast would the rider be traveling as she enters the irregular loop?

10 How many g's does the rider feel as she enters the irregular loop?

11 How many g's does the rider feel as she enters the circular loop?

12 How many g's does the rider feel as she passes over the top of the circular loop?

ANSWERS

1	15.65 m/s	2	35.00 m/s	3	6 g's	4	35 m/s
5	3.5 g's	6	16.65 m/s	7	1.00 g's	8	19.17 m/s
9	36.71 m/s	10	3.75 g's felt	11	6.50 g's felt		

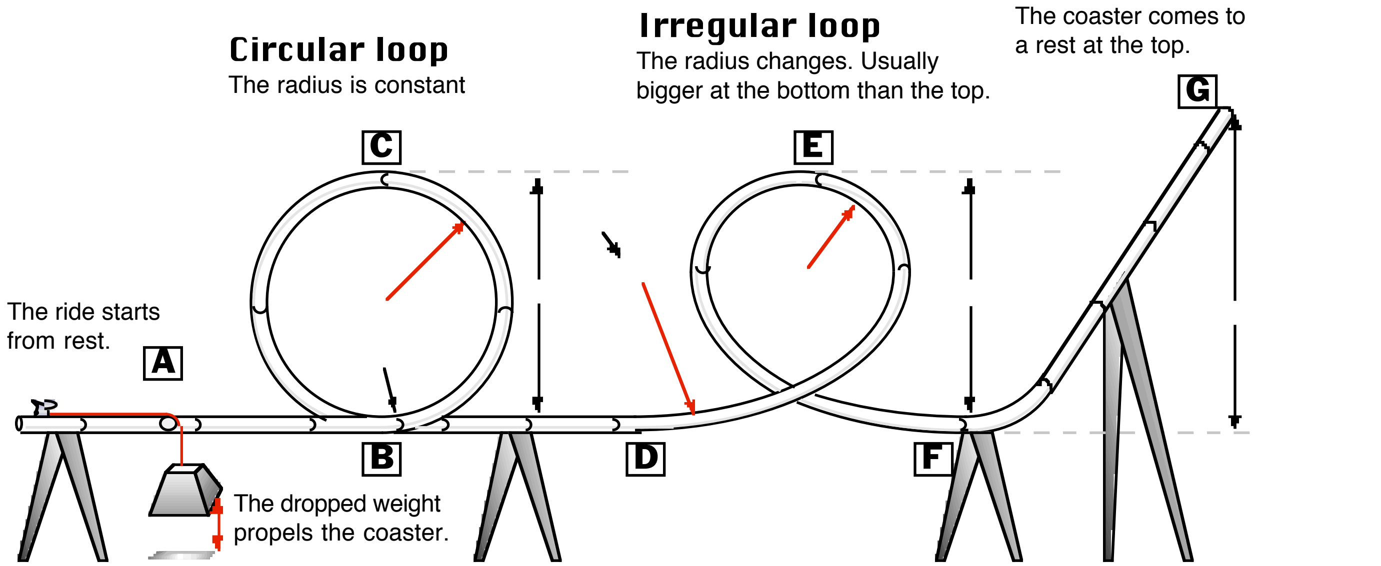
12 0.50 g's (He feels like he might fall out of his seat.)

 by Tony Wayne

The activity on the following page is good for a quick introduction to loop design. It is appropriate for students who lacks the necessary math skills. It could also be used as a quick overview to loop design.

The first page is to be used as a reference. The second page is where the calculations are done on a spreadsheet.

This diagram is to be used in conjunction with the spreadsheet below and the questions on the following page.



Below is the spreadsheet and its formulae for the “Investigating the Loop Using a Spreadsheet” handout. The text on the left hand side is in the “B” column. The “B” column is right justified.

The Spreadsheet’s formulae

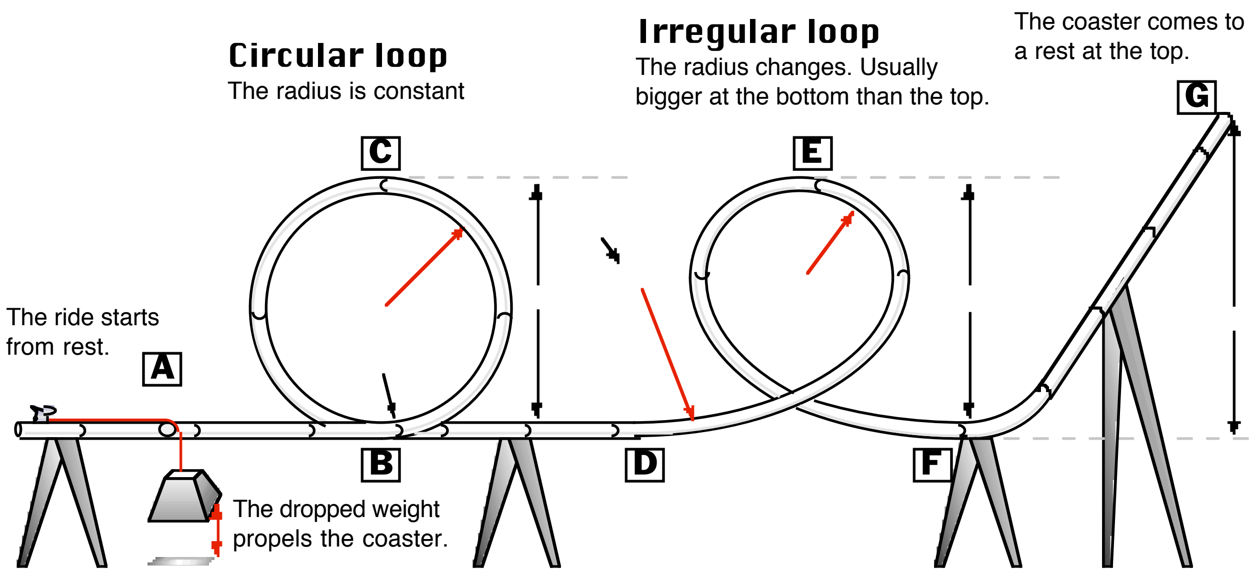
	A	B	C
1		train's mass (kg):	
2		weight's mass (kg):	
3		weight's drop height (m):	
4		Train's Acceleration (g's):	=(C2/(C3+C2))
5		Velocity at "A" (m/s):	=SQRT(2*C4*9.8*C3)
6		Velocity at "B" (m/s):	=C5
7		Radius of 1st loop (m):	
8		g's felt at "B":	=(C6*C6/C7)/9.8+1
9		Height at "C" (m):	=C7*2
10		Velocity at "C" (m/s):	=SQRT(C6*C6-(2*9.8*C9))
11		g's felt at "C"	=(C10*C10/C7)/9.8-1
12		Velocity at "D" (m/s):	=C5
13		Radius at "D" (m):	
14		g's felt at "D":	=(C12*C12/C13)/9.8-1
15		Radius at "E" (m):	
16		Height at "E" (m):	
17		Velocity at "E" (m/s):	=SQRT(C6*C6-(2*9.8*C16))
18		g's felt at "E":	=(C17*C17/C15)/9.8+1
19		Height to "G" (m):	=C12*C12/19.6

Questions:

1 A 5500 kg coaster train is propelled by a 120,000 kg weight that is dropped 20.0 m to the ground. The first loop has a radius of 25 m.
   a) How many g's are felt by the rider as he enters the first loop?
   b) How fast is the rider traveling as he travels over the top of the first loop?
   c) How many g's are felt by the rider as he travels over the top of the first loop?
   d) Make the radius at the bottom of the irregular loop 25 m. What must the radius at the top of the second loop be if its height is 42 meters?

2 A 5500 kg coaster train is propelled by a 91,000 kg weight that is dropped 25.0 m to the ground.
   a) What must the radius of the first loop be so that a rider feels 2 g's as she enters the loop?
   b) What must the radius of the second loop be so that a rider feels 2 g's as she enters the loop?
   c) How high and what radius must the irregular loop be so that a rider feels the same g's at the top and bottom?

3 Design a roller coaster where the rider feels 2.9 to 3.1 g's at every acceleration *except* at the top of the first loop. Enter your numbers at the appropriate locations on the diagram below.



4 In terms of g's felt by a rider, what are the benefits of using an irregular loop versus a circular loop?

Center of Mass

CENTER OF MASS

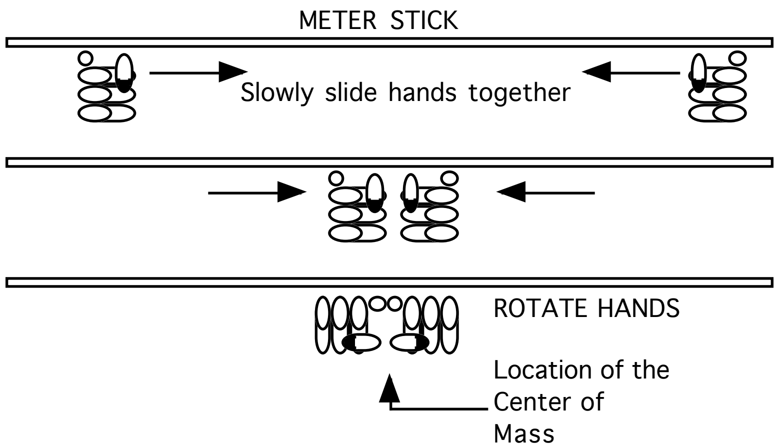
Center of Mass
The balance point of an object or collection of objects.

DEMO

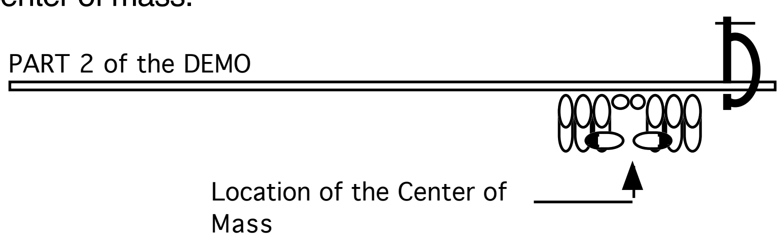
Materials: Meter stick, C-clamp

Procedure

Hold the meter stick horizontally between two fingers. Slowly slide your hands together. For a real challenge, close your eyes when sliding your hands together. Your hands will always meet under the center of mass.



Attach a c-clamp at one end of the meter stick. Redo the demonstration. Your fingers will still meet under the center of mass.



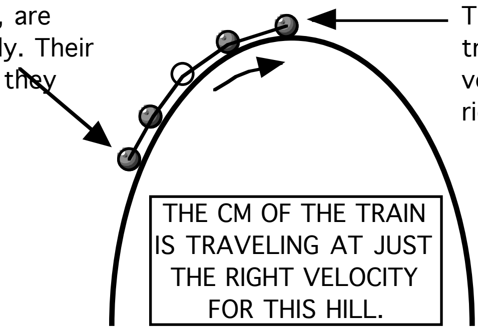
CENTER OF MASS OF A ROLLER COASTER TRAIN

The center of mass of a train would be in the center of the train. This is assuming all the riders are of the same mass. The feeling a track is designed for is engineered around the center of mass of a train. A rider gets a different feeling if she is to ride some distance away from the center of mass.

OVER THE HILLS

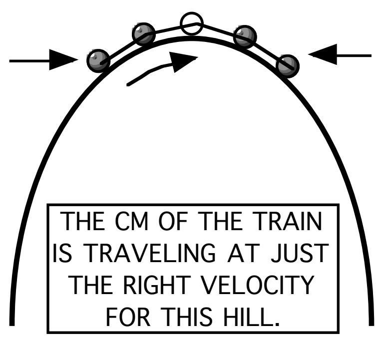
A hill is designed for a specific velocity. The design velocity is chosen such that the rider located at the train's center of mass will, at most, feel weightless. The hill's shape determines the design velocity. This shape also dictates a specific velocity at each part of the hill.

These cars, in the back, are slowing down too quickly. Their riders feel held back as they approach the hill.



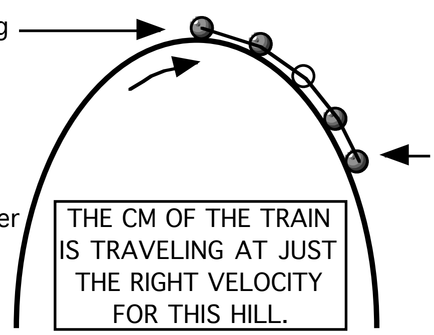
These cars, in the front, are traveling faster than the design velocity for the hill's peak. These riders feel pushed over the hill.

The cars in back are traveling too slow for their position on the hill. The riders feel held back. They feel like they are hanging on the hill.



The cars in front are traveling too slow for their position on the hill. The riders feel held back. They feel like they are hanging on the hill.

The cars in back are traveling faster than the hill's design velocity. The riders feel whipped out of their seat prematurely due to the negative g's. The shoulder or lap harness is holding the rider in place.



The cars in front are traveling too slow for their position on the hill. The riders feel less of a weightless sensation.

This can be demonstrated by using the HotWheels™ train. (Construction of the HotWheels™ train is described on page 81.) Set up a box with a track running horizontally over the top. Slowly roll the train over the hill. As the front of the train begins to pass over the hill it will not speed up until the middle, center of mass of the train, travels over the hill.

Banked Curves

(Circular Motion, Free Body Diagrams, Kinematics and Physiological Effects)

HORIZONTAL CURVES AND TURNS

A horizontal curve is a curve that does not rise or fall. There are two type of curves, flat curves and banked curves.

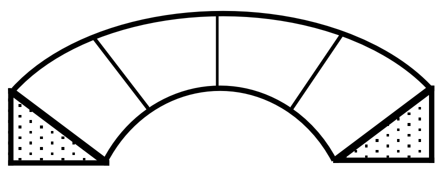
FLAT CURVES

A flat curve gives a rider the sensation of being thrown sideways. If the roller coaster car's velocity is fast enough and the radius small enough, the stresses on the car's under carriage can be tremendous. For a flat curve the inward net acceleration felt by the rider is calculated from the equation.

$$a = \frac{v^2}{R}$$

Where "a" is the acceleration felt by the rider to the inside of the circle, "v" is the velocity of the car and "R" is the radius of the curve. This acceleration can be converted to g's by dividing it by 9.80 $^{m}/_{s^2}$.

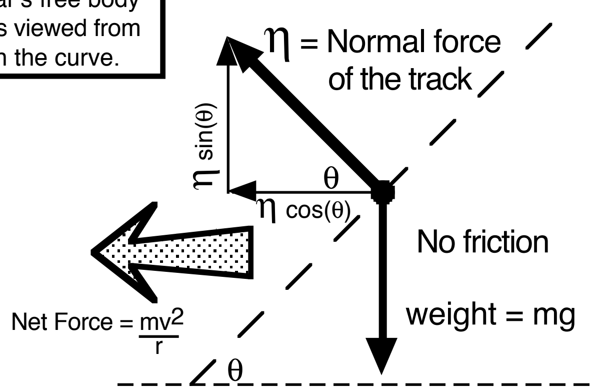
BANKED CURVES



A banked curve reduces the rider's sensation of being thrown sideways by turning the car sideways. The car is tilted. The trick is to tilt the track just the right amount.

The ideal banked curve is one where no outside forces are needed to keep the car on the track. In other words, if the banked curve were covered with ice -no friction- and the coaster did not have a steering mechanism the car would stay on the track. These are the forces acting on the car as the car travels around horizontal banked curves.

Coaster car's free body diagram as viewed from the rear on the curve.



This diagram yields the following relationships

$\Sigma F_x = mv^2/r = \eta \cos(\theta)$

$\Sigma F_y = 0 = \eta \sin(\theta) - mg$

therefore

from $\Sigma F_x \Rightarrow \quad \eta = \dfrac{mv^2}{(R)\cos(\theta)}$

from $\Sigma F_y \Rightarrow \quad \eta = \frac{mg}{\sin(\theta)}$

$$\therefore \quad \frac{mg}{\sin(\theta)} = \frac{mv^2}{(R)\cos(\theta)}$$

$$\frac{g}{\sin(\theta)} = \frac{v^2}{(R)\cos(\theta)}$$

$$R = \frac{v^2 \sin(\theta)}{g\cos(\theta)}$$

$$\boxed{R = \frac{v^2 \tan(\theta)}{g}}$$

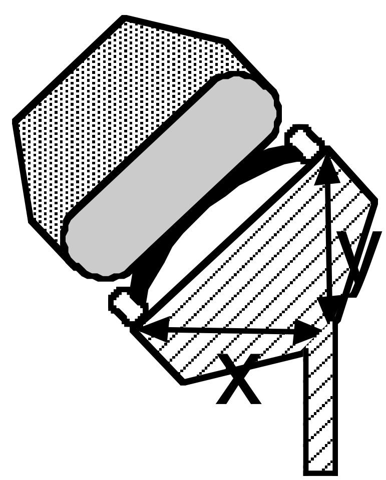
This is for the *ideal* banked curve where no friction is required to keep the car from sliding to the outside or inside of the curve. On a given curve if the velocity is greater or less than the design velocity then the cars may need a little frictional help to keep them on the track.

If your not comfortable with trigonometry functions, the equations can be rewritten and used as shown below.

$$\mathbf{a = \frac{(g)y}{x}}$$

$$\mathbf{v = \sqrt{\frac{(Rgy)}{x}}}$$

$$\mathbf{R = \frac{(x)v^2}{(g)y}}$$



The draw back to this method is in measuring the lengths of "x" and "y."

Calculating g's Felt on a Banked Curve

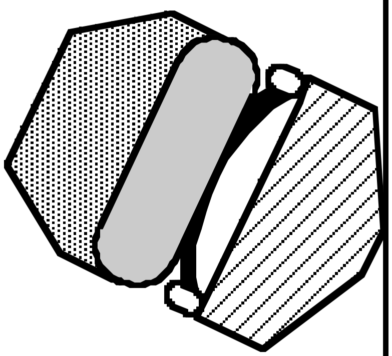
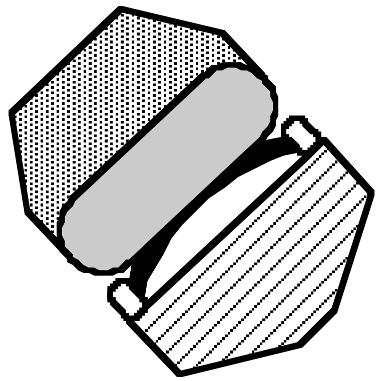
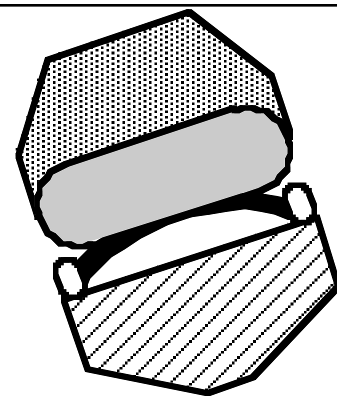
Recall that the g's felt is equal to the normal force divided by mass and then divided by g to convert to from m/s2 to g's.

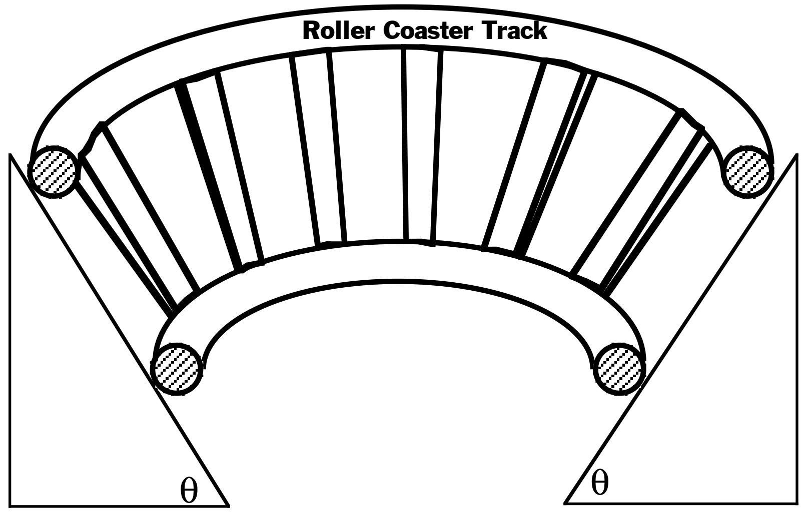
from $\Sigma F_y \Rightarrow \quad \eta = \frac{mg}{\sin(\theta)}$ ... from the above derivation.

g's felt $\Rightarrow \quad \eta = \frac{\cancel{mg}}{\sin(\theta)\cancel{mg}}$

$$\text{g's felt} = \frac{1}{\sin(\theta)}$$

Remember this is for the ideal banked curve with no friction.

HIGH SPEED BANK	MEDIUM SPEED BANK	LOW SPEED BANK
		
Too much bank for the car's velocity. the car could tip to the inside. The undercarriage wheels are holding the car on. The rider fells a force pushing himself down. Friction is needed to keep the car on the track.	At just the right bank for the car's velocity, the car does not need any type of undercarriage to stay on the track. The rider feels a force pushing his bottom into the seat. This is the optimum position where no friction is needed to keep the car on the track.	Not enough bank for the car's velocity. The car could tip to the outside. The undercarriage wheels are holding the car on. The rider fells a force pushing himself to the outside of the curve -sideways. Friction is needed to keep the car on the track.



$$\text{g's felt} = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{v^2}{rg}$$

θ is the angle where no outside forces other than gravity are needed to keep the car from sliding to the outside or inside of the curve.

1 What must the curve's angle be for a roller coaster car to travel around a curve of radius 30 m at 20 m/s?

2 How many g's are felt by a rider as he travels around the banked curve in the previous problem?

3 A car is to make it around a banked curve. The radius is 15.35 m and the car will travel at 30 m/s. What is the optimum banking angle of the curve?

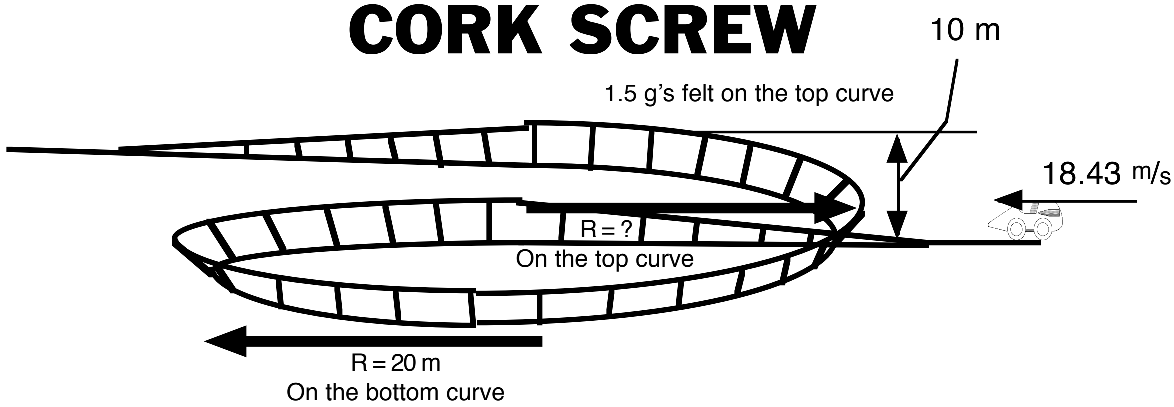
4 A car is to make it around a banked curve. The radius is 15.35 m and the car will travel at 30 m/s. This roller coaster is on the moon where the acceleration due to gravity is 1.67 m/s^2. What is the optimum banking angle of the curve?

5 A rider is to make it around a curve of radius of 24.28 m so that the rider will feel 2.50 g's. What is the angle of the banked curve?

6 A rider is to make it around a curve of radius of 31.15 m so that the rider will feel 1.64 g's. How fast must the rider be traveling?

7 A rider is to make it around a curve of radius of 51.15 m so that the rider will feel 4.52 g's. How fast must the rider be traveling?

CORK SCREW



8 What is the banked angle of the bottom curve?

9 How many g's are felt by the rider along the bottom curve?

10 What is the optimum angle of the top banked curve after spiraling up 10 m?

11 What is the radius of the top curve?

ANSWERS

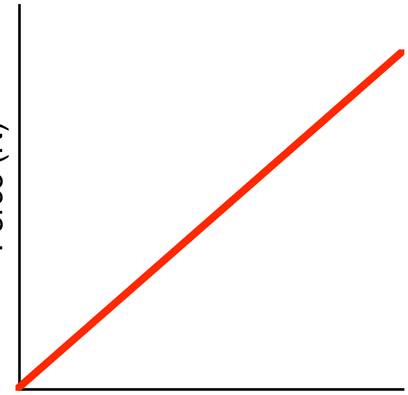
1	53.68°	2	1.69 g's	3	80.51°	4	88.37°
5	66.42°	6	19.92 m/s (52.69°)	7	47.01 m/s (77.22°)	8	60.01°
9	2.000 g's	10	48.19°	11	13.11m (11.99 m/s at the top)		

SPRINGS

There are two main formulas for modeling the behavior of a spring. The force is found from

$$F = -kx$$

Where “k” is the force constant for the spring. It is measured in N/m. “x” is the displacement from equilibrium in meters. “F” is te force in Newtons. The negative sign is in this equation because the force always opposes the direction the spring is stretched. When calculating the magnitude of the force, drop the negative sign. A graph of force versus displacement looks like the one below.



This means the force is not constant but varies with time. This also means that the acceleration changes as the spring’s displacement changes. To calculate the MAXIMUM acceleration of an object stuck on the spring look at the spring when the spring is compressed the most and set this equal to Newton’ second law.

$$F_{spring} = F_{Newton}$$

$$k(x_{max}) = ma_{max}$$

“k” is the spring constant and x_{max} is the distance the springis compressed, “m” is the train’s mass and a_{max} is the acceleration of the train. If the final acceleration is zero, once it is released from the spring, then the average acceleration is half the max. Your project limits the avearge acceleration to 5.5 g’s. This means the maximum acceleration is 11 g’s, which is 107.8 m/s^2. Therefore,

$$k(x_{max}) = m(107.8)$$

You, the designer, need to pick a combination of displacement and spring constant that meets the constraints above. Then put these numbers in the energy equations for other calculations.

When an object hits the spring, the spring removes the energy from body. The energy stored in the spring comes from finding the work under the curve above.

$$E_{spring} = \frac{1}{2}kx^2$$

This is a potential energy, just like gravitational potential energy. It can be equated to kinetic energy.

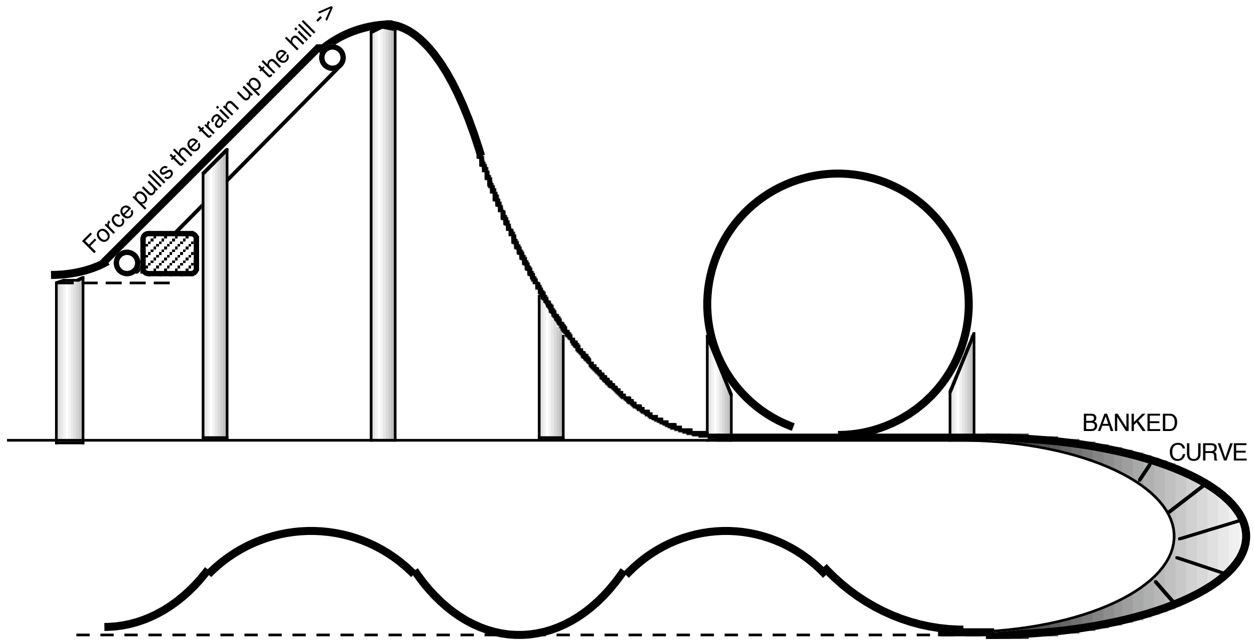
The displacement comes from the earlier calculation. If you need more energy, redo your earlier numbers and make the displacement and greater and the spring constant smaller. This will give you more energy because the displacement in the energy formula is squared and the spring constnat is not.

Introduction to Roller Coaster Design (Example)

Intro to Design Example

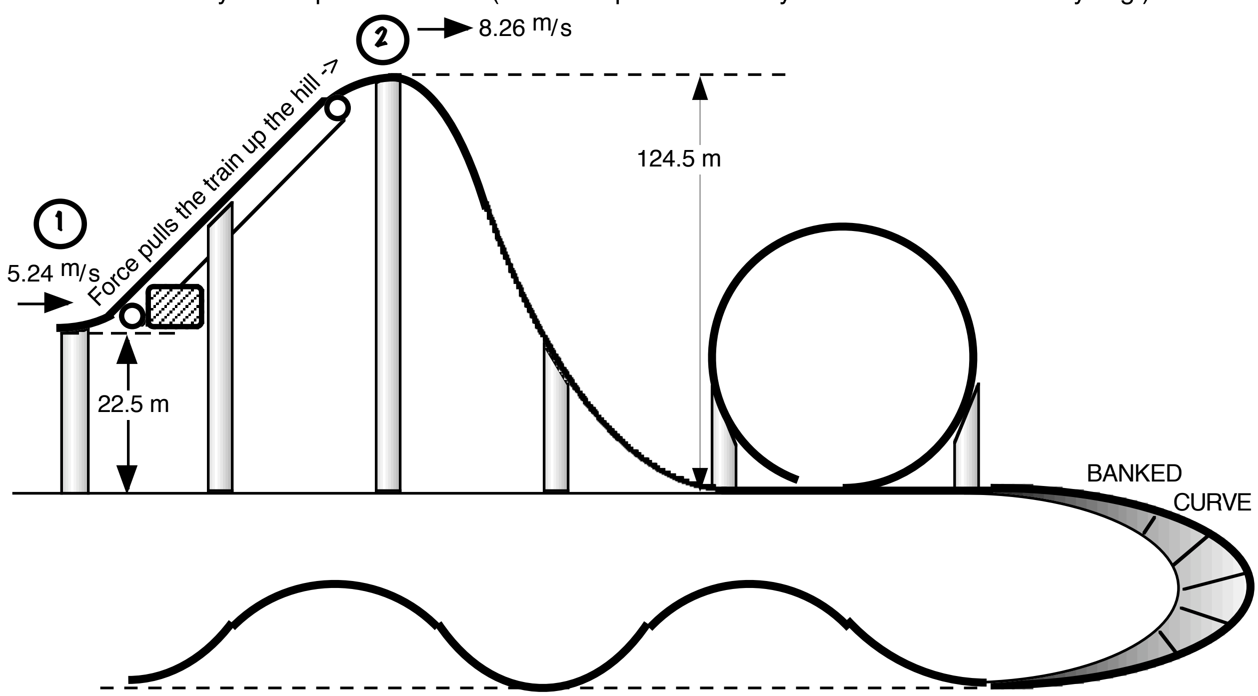
This will show the reader the basic steps to designing a roller coaster. The example coaster will not be the best possible design. (I don't want students to use it as their own in other projects.)

STEP 1 Draw a picture of what the coaster *may* look like.



STEP 2 Assign some beginning numbers.

Numbers like the initial velocity as the coaster train leaves the station. The mass of the coaster train. And/or an initial velocity as it tops the first hill. (Label the pieces for easy identification when analyzing.)

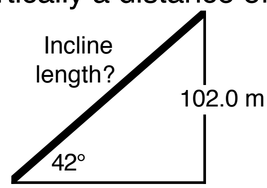


STEP 3 Begin to calculate everything you can and check to see if it makes sense.

For this design start by calculating the force needed to pull the train up the incline and the power to pull it up the incline.

In order to calculate the power force and power to pull it up the incline I'm going to need the train's mass. So make up a reasonable mass. This coaster is made up of 6 cars. Each car has a maximum mass, with two riders at 100 kg each, of 735 kg; (535 kg car + 100 kg rider + 100 kg rider.) Therefore, the coaster train will have a mass of 4410 kg.

What angle will the first incline be? The designer can choose this number too. 42° is good. The train is going to be pulled up vertically a distance of, (124.5-22.5), 102.0 m.



$$\text{Incline length} = 102/(\sin 42°)$$
$$\text{Incline length} = 152.4 \text{ m}$$

$$ET_{(\text{OUT OF STATION})} + \text{Work} = ET_{(\text{TOP OF } 1^{st} \text{ HILL})}$$
$$KE + PE + W = KE + PE$$
$$(1/2)mv^2 + mgh + Fd = (1/2)mv^2 + mgh$$
$$(1/2)4410(5.24)^2 + 4410(9.8)(22.5) + F(152.4) = (1/2)4410(8.26)^2 + 4410(9.8)(124.5)$$
$$60544.008 + 972405 + F(152.4) = 150441.858 + 5380641$$
$$F(152.4) = 4498133.85$$

$\underline{F = 29515.314 \text{ N}}$ ... is the pulling force along the incline.

How much time will it take to travel up the incline?

The acceleration of the train is found from

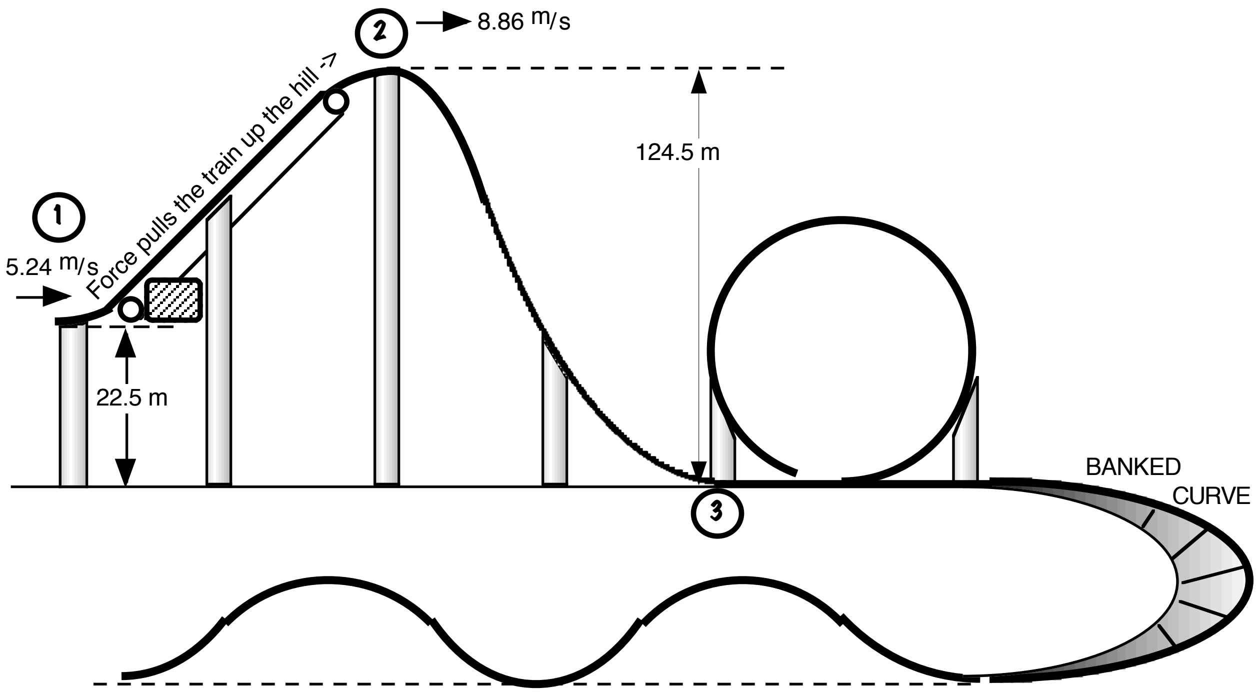
$$(v_f)^2 = (v_o)^2 + 2ad$$
$$(8.26)^2 = (5.24)^2 + 2(a)d$$
$$a = 0.134 \text{ m/s}^2$$

$$v_f = v_o + at$$
$$8.26 = 5.24 + 0.134(t)$$

$\underline{t = 22.537 \text{ sec}}$ ...is the time to climb the incline. (Most initial lift times are between 60 and 120 seconds.)

It is beyond the scope of this book to show how to calculate the time for each track element. It was shown here because of its ease of calculation.

STEP 4 Calculate maximum velocity of the ride.



Since the 1st drop is the longest, the velocity at the bottom will be the greatest, (location #3). Energy relationships will be used to calculate the velocity.

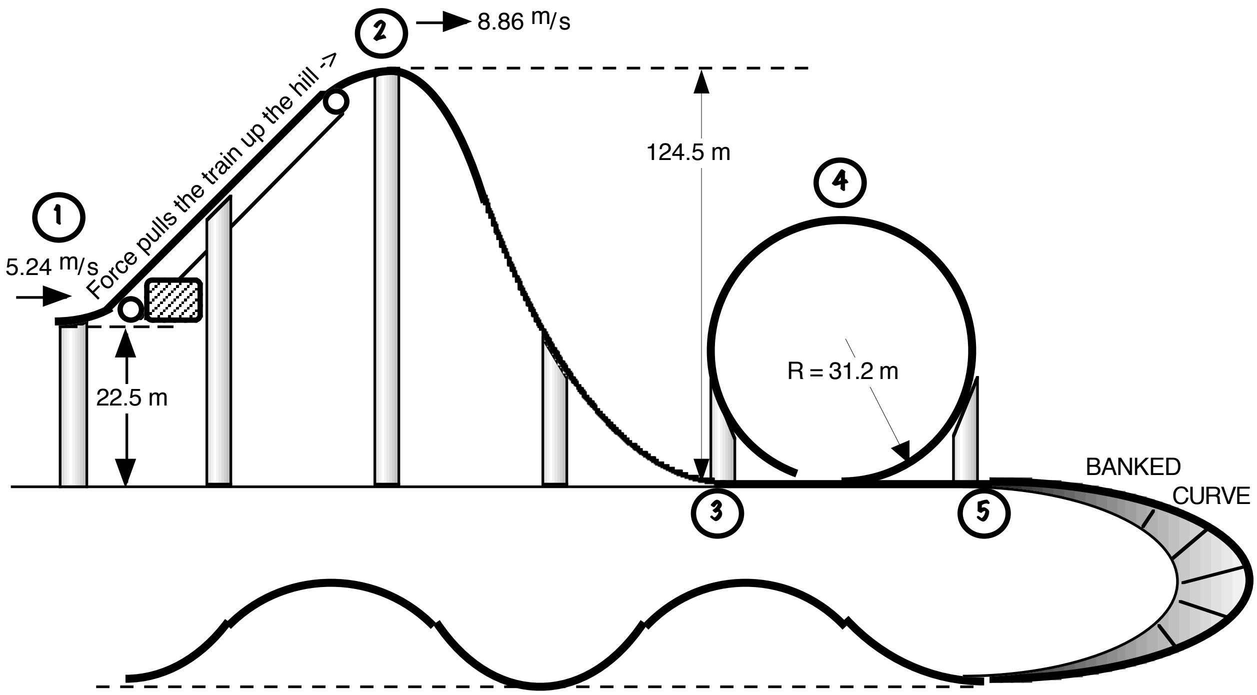
$$ET_{(LOCATION \#2)} = ET_{(LOCATION \#3)}$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$
$$(1/2)4410(8.26)^2 + 4410(9.8)(124.5) = (1/2)4410(v)^2 + 4410(9.8)(0)$$
$$150441.858 + 5380641 = 2205(v)^2$$
$$2508.428 = (v)^2$$
$$v = 50.084$$

$\underline{v = 50.1}$ m/s ... At the bottom of the first hill

That's 112 mi/hr !!!

STEP 5 The loop.

For any loop, the designer would like to know the velocity as the rider enters the loop; at the top of the loop; and as the rider leaves the loop. The designer would also like to know the g's felt by the passengers. This is the location on the ride where riders are most likely to pass out if the g's are too much. The radius in the loop below is made up.



The velocity as the rider enters the loop and as the rider leaves the loop is the same as the velocity at the bottom of the first hill. This is because all three locations are at the same height.

The velocity at the top of the loop is not the same as at the bottom. As the coaster travels up the loop it will lose kinetic energy and gain potential energy.

The height of the loop is simply double the radius. h = 2(31.2) = 62.4 m

$$ET_{(LOCATION\ \#2)} = ET_{(LOCATION\ \#4)}$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + mgh = (1/2)mv^2 + mgh$$
$$(1/2)4410(8.26)^2 + 4410(9.8)(124.5) = (1/2)4410(v)^2 + 4410(9.8)(62.4)$$
$$150441.858 + 5380641 = 2205(v)^2 + 2696803.2$$
$$1285.388 = (v)^2$$
$$v = 35.852$$
$\underline{v = 35.9}$ m/s ... At the bottom of the first hill
That's 80.3 mi/hr !!!

If you are doing this calculation and you get an expression that requires you to calculate the velocity by taking the SQUARE ROOT OF A NEGATIVE NUMBER, then the loop is too tall for the given velocity at the bottom of the loop. The velocity will need to be increased or the height of the loop decreased.

To calculate the g's felt by the rider, calculate the centripetal acceleration at each location, convert to g's and either add or subtract a g as necessary.

As the rider enters and leaves the loop

v = 50.084 m/s

r = 31.2 m

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{50.084^2 \text{ m/s}}{31.2 \text{ m}}$$

$$a_c = 80.398 \text{ m/s}^2$$

$$a_c = \frac{80.398 \text{ m/s}^2}{9.80 \text{ m/s}^2}$$

$a_c = 8.2$ g's

$a_c = 8.2$ g's + 1g

$a_c = 9.2$ g's ... That is an incredible amount of g's. Most coasters do not go above 5 g's. To be safe the radius at the bottom of the loop needs to be bigger.

As the rider passes the top of the loop

v = 35.852 m/s

r = 31.2 m

$$a_c = \frac{v^2}{r}$$

$$a_c = \frac{35.852^2 \text{ m/s}}{31.2 \text{ m}}$$

$$a_c = 41.198 \text{ m/s}^2$$

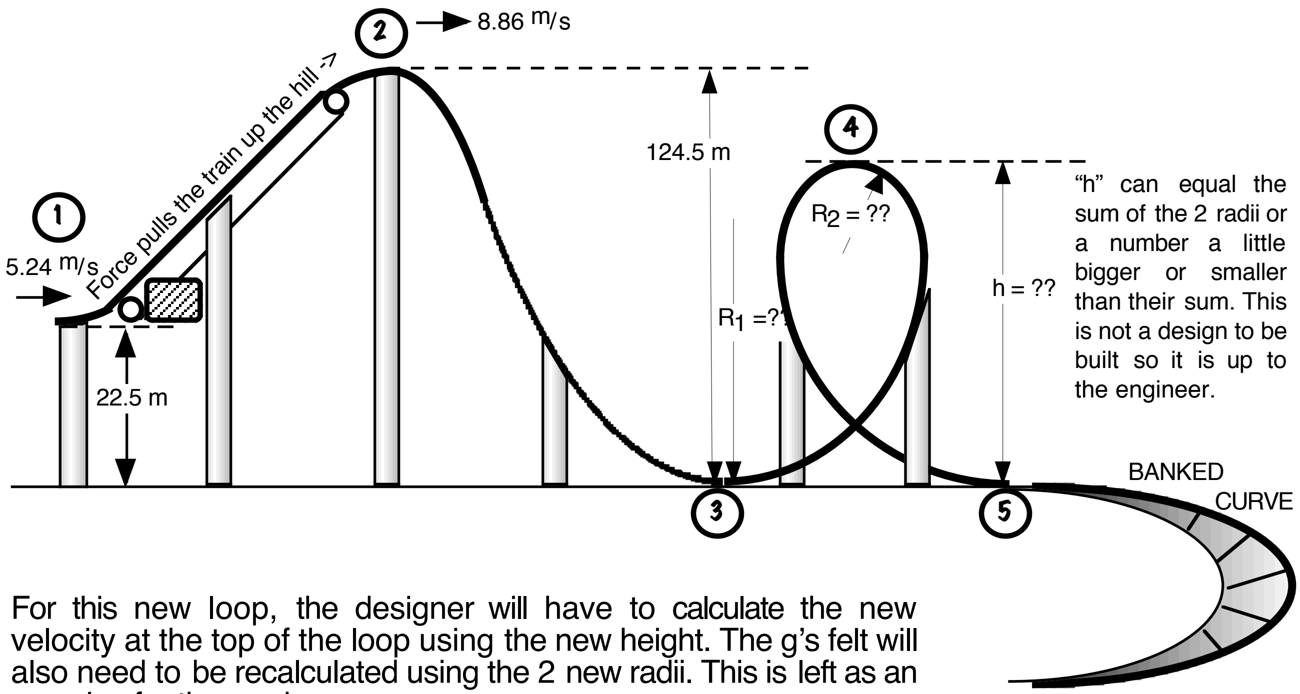
$$a_c = \frac{41.198 \text{ m/s}^2}{9.80 \text{ m/s}^2}$$

$a_c = 4.2$ g's

$a_c = 4.2$ g's - 1g

$a_c = 3.2$ g's ... That is an acceptable amount. But 3.2 g's is rather high for the top of a loop. Most of the time the g's at the top of a loop are from 1.5 to 2 g's.

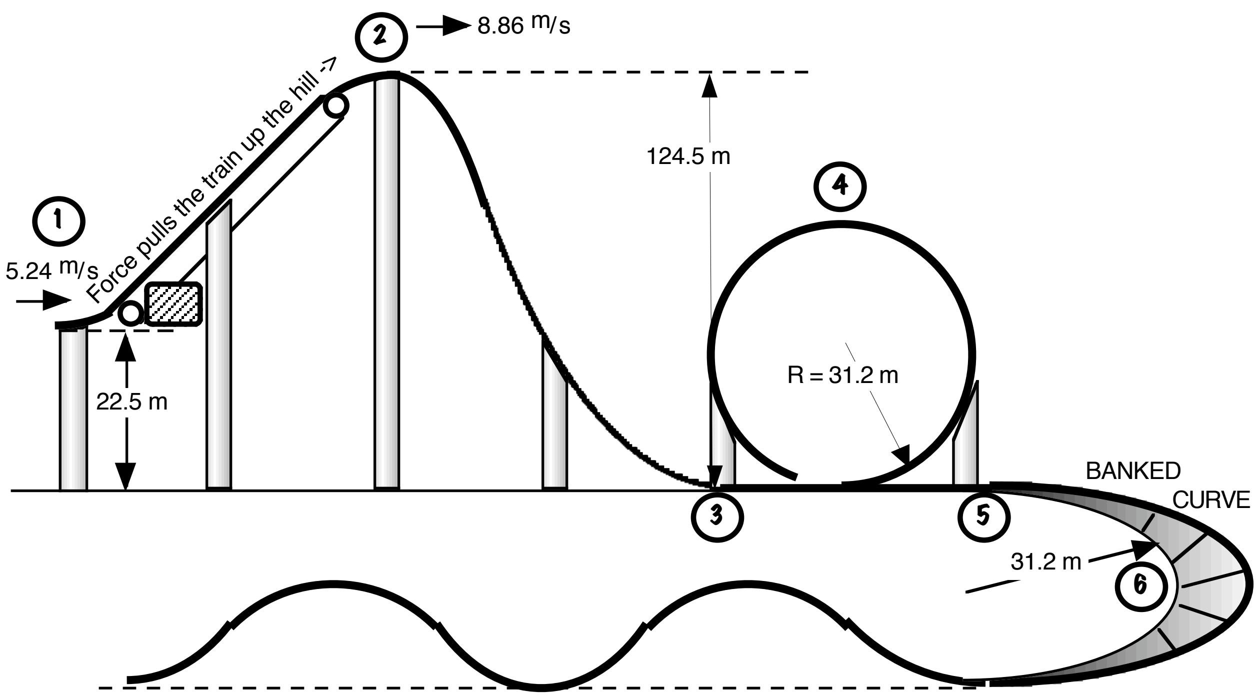
Because of the need for a larger radius as the rider enters the loop above, this coaster might be a good candidate for an irregular loop like the one below.



For this new loop, the designer will have to calculate the new velocity at the top of the loop using the new height. The g's felt will also need to be recalculated using the 2 new radii. This is left as an exercise for the reader.

STEP 6 The banked curve.

The banked curve is a horizontal curve on the ground in this diagram. Because it is at the lowest point it's velocity is equal to that of location #3, where v = 50.084 m/s. For a first try I'll make the radius 31.2 m.



The curve will be designed at the optimum angle where no friction or outside lateral forces are needed to keep the car on the track at speed. "At speed" means the velocity of the track design.

$\tan(\theta) = \frac{v^2}{rg}$

$\tan(\theta) = \frac{50.084^2 \text{ m/s}}{31.2 \text{ m}(9.8) \text{ m/s}^2)}$

$\tan(\theta) = 0.164$

$\theta = 9.3°$ .... That's almost a flat turn. It might be more exciting to try to decrease the radius so a greater banking angle will be needed.

The g's felt are calculated from

g's felt $= \frac{1}{\cos(\theta)}$

g's felt $= \frac{1}{\cos(9.3°)}$

g's felt = 1.013 g's .... This is not much more than normal gravity. These g's are the g's felt applied to your seat. Because this curve is rather flat, it would be wise to examine lateral, centripetal, acceleration in g's.

Lateral g's are the g's felt in the horizontal plain of the curve.

v = 50.084 m/s

r = 31.2 m ... of the curve. This just happens to be the same as the loop's radius.

$a_c = \frac{v^2}{r}$

$a_c = \frac{50.084^2 \text{ m/s}}{31.2 \text{ m}}$

$a_c = 80.398 \text{ m/s}^2$

$a_c = \frac{80.398 \text{ m/s}^2}{9.80 \text{ m/s}^2}$

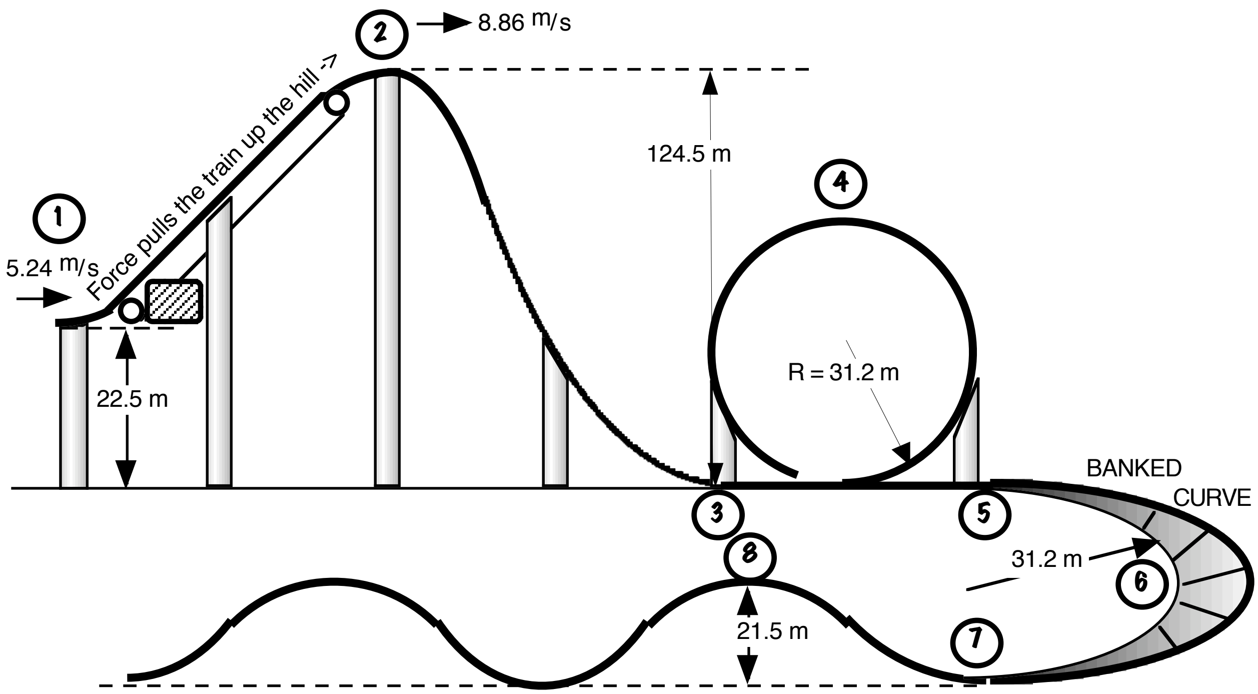
$a_c = 8.2$ g's ... Do not add or subtract a "g" because the circular motion is horizontal and not in the vertical plain.

... That is a lot of lateral g's. That's 8.2 times the rider's weight pressing him against the side of the coaster car. It is too extreme. Maybe a value around 1 to 2 g's would be better tolerated by the rider.

The banked curve needs to be redesigned.

STEP 7 The camel back

The camel back humps begin at the lowest part of the track and climb to a height of 21.5 m. The calculations to check the velocity at the top of the hump is similar to the one for the drop from location 2 to 3.



$$ET_{(LOCATION\ \#7)} = ET_{(LOCATION\ \#8)}$$
$$KE + PE = KE + PE$$
$$(1/2)mv^2 + 0 = (1/2)mv^2 + mgh$$
$$(1/2)v^2 + 0 = (1/2)v^2 + gh$$
$$(1/2)(50.084)^2 + 0 = (1/2)(v)^2 + (9.8)(21.5)$$
$$1254.203528 = (1/2)(v)^2 + 210.7$$
$$2087.007056 = (v)^2$$
$$v = 45.6838$$

$\underline{v = 45.7}$ m/s ... At the bottom of the first camel back hill.

That's 102 mi/hr !!!

It might be wise to decrease the height of the first drop to decrease the velocity of the coaster as it enters the loop and the banked curve. Then, redesign the loop and banked curve to reduce g's experienced by the rider. As for the camel back humps at the end of the ride, they probably need to either be taller or stretched out horizontally. They are too narrow, horizontally, for 45.7 m/s.